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# NAVAL POSTGRADUATE SCHOOL

Monterey, California



## THESIS

A COMPARATIVE STUDY ON THE REALIZABILITY  
OF BIQUADRATIC FUNCTIONS

by

Achmad Ischak Suparman

June 1976

Thesis Advisor:

Shu-gar Chan

Approved for public release; distribution unlimited.

Thesis  
S8639

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A Comparative Study on the Realizability  
of Biquadratic Functions

by

Achmad Ischak Suparman  
Lieutenant Colonel, Indonesian Air Force  
B.S., Posts & Telecommunications Academy  
Bandung, Indonesia, 1963

Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL  
June 1976

### ABSTRACT

A comparative study is made on the realizability conditions of various special cases as well as the general case of the positive-real biquadratic functions. Computer programs are written for this study to illustrate the realizability regions of zeros (or poles) of the functions. A limited study is also made on the sensitivity of the function to variation of element value in a realization.

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## I. INTRODUCTION

The problem of realizability of biquadratic functions as driving point immittance has been studied by a number of investigators [1] - [11], with realizability conditions given for various special cases as well as for the general case of biquadratic functions. It has been proved [10] that, given a pair of poles, the location of the zeros are limited by two curves, one of which is the locus of the zeros such that the biquadratic function is a minimum positive real function.

In Chapter II of this paper, various realizability conditions are reviewed and illustrated. FORTRAN programs are written in Chapter III for illustration as well as comparison of different cases.

Many forms of realizations of positive-real biquadratic functions are well known. Some of these forms are utilized, in Chapter IV, to study the sensitivity problem related to the realizability of the function. The method of graphical representation by Chan [13] is applied in this study. A computer program is written for the illustration of the variation of sensitivity phasors [13] as functions of element values.

Finally, in Chapter V, a discussion on the results is made together with some suggestions for further studies.

## II. REALIZABILITY CONDITIONS

### A. INTRODUCTION

Consider a general biquadratic function

$$Z(s) = \frac{s^2 + 2\sigma_z s + \omega_z^2}{s^2 + 2\sigma_p s + \omega_p^2} = \frac{(s + z')(s + z'')}{(s + p')(s + p'')} \quad (1)$$

or

$$Z(j\omega) = \frac{-\omega_z^2 + 2\sigma_z j\omega + \omega_z^2}{-\omega_p^2 + 2\sigma_p j\omega + \omega_p^2} = \frac{(\omega_z^2 - \omega^2) + j\omega 2\sigma_z}{(\omega_p^2 - \omega^2) + j\omega 2\sigma_p} \quad (2)$$

It has been investigated by Chan and Phung [10], that a biquadratic function  $Z(s)$  as given in equation (1) with conjugate-complex zeros in the open half plane is realizable as a driving point immittance with passive one-port if and only if the zeros are located either on the curve  $C_0$  or  $C_1$  or inside the area delimited by curves  $C_0$  and  $C_1$ , whose polar equations are given respectively by

$$\max \omega_z = \omega_p + 2\sigma_p \cos \phi_z' + 2 \left[ \sigma_p \cos \phi_z' (\sigma_p \cos \phi_z' + \omega_p) \right]^{\frac{1}{2}} \quad (3-a)$$

and

$$\min \omega_z = \omega_p + 2\sigma_p \cos \phi_z' - 2 \left[ \sigma_p \cos \phi_z' (\sigma_p \cos \phi_z' + \omega_p) \right]^{\frac{1}{2}} \quad (3-b)$$

where  $\phi_z'$  denotes the compliment of the argument of the zero  $z'$ , such

that

$$\sigma_z = \omega_z \cos \phi_z'.$$

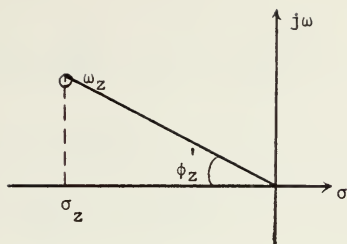


Figure 1

#### B. NECESSARY AND SUFFICIENT CONDITIONS

To obtain the real part of  $Z(j\omega)$  of equation (2), multiply and divide this equation with the conjugate of the denominator, so that

$$\operatorname{Re} [Z(j\omega)] = \frac{(\omega_p^2 - \omega^2)(\omega_z^2 - \omega^2) + 4\sigma_p\sigma_z\omega^2}{(\omega_p^2 - \omega^2)^2 + 4\sigma_p^2\omega^2}. \quad (5)$$

$Z(s)$  is positive real if the numerator of expression (5) is nonnegative for all values of  $\omega$ , that is

$$N(\omega^2) = \omega^4 - \omega^2(\omega_z^2 + \omega_p^2 - 4\sigma_z\sigma_p) + \omega_z^2\omega_p^2. \quad (6)$$

Now we have the following two conditions [10]:

##### Condition-1

$N(\omega^2)$  has complex or equal positive real roots.

##### Condition-2

$N(\omega^2)$  has nonpositive real roots.

Condition-1 implies the discriminant of  $N(\omega^2)$  is negative or zero

i.e.

$$D = (\omega_z^2 + \omega_p^2 - 4\sigma_z\sigma_p)^2 - 4\omega_z^2\omega_p^2 \leq 0 \quad (7)$$

or after factoring we get

$$\left[ (\omega_z + \omega_p)^2 - 4\sigma_z \sigma_p \right] \left[ (\omega_z - \omega_p)^2 - \sigma_z \sigma_p \right] \leq 0 ; \quad (7-a)$$

thus condition-1 is satisfied if

$$(\omega_z - \omega_p)^2 \leq 4\sigma_z \sigma_p < (\omega_z + \omega_p)^2 . \quad (7-b)$$

The right hand inequality is shown as strict since inequality corresponds to real roots which would in this case be nonpositive, which is a violation of condition-1.

Next, condition-2 implies the following two conditions simultaneously:

Condition-2a

$$D \geq 0 .$$

Condition-2b

$$\omega_z^2 + \omega_p^2 - 4\sigma_z \sigma_p \leq 0 .$$

It can be seen that both conditions 2a and 2b are satisfied if

$$4\sigma_z \sigma_p \geq (\omega_z + \omega_p)^2 . \quad (8)$$

With equations (7-b) and (8) we obtain the necessary condition for positive realness of  $Z(s)$  to be

$$(\omega_z - \omega_p)^2 \leq 4\sigma_z \sigma_p . \quad (9)$$

Now suppose that equation (9) is satisfied with the equality sign.  $Z(s)$  is then a minimum positive-real function since  $N(\omega^2)$  has a double positive root  $\omega_i$ , where

$$\omega_i^2 = \omega_z \omega_p . \quad (10)$$

Next suppose equation (9) is satisfied but not necessary as an equality, then

$$\sigma_z \geq \frac{(\omega_z - \omega_p)^2}{4\sigma_p} \quad (11)$$

The necessary and sufficient condition obtained in the above discussion can be conveniently and geometrically interpreted.

Suppose that the zeros  $z'$  and  $z''$  are conjugate complex with negative real parts and the poles  $p'$  and  $p''$  are similarly located in the open left half plane.

Let

$$\sigma_z = \omega_z \cos \phi_z' \quad (12-a)$$

where  $\phi_z'$  denotes the compliment of the argument of the zero  $z'$ .

$$\text{If } \omega_z \geq \omega_p, \text{ then we let } \omega_z = \omega_p + u_0 \quad (12-b)$$

$$\text{If } \omega_z \leq \omega_p, \text{ then we let } \omega_z = \omega_p - u_1 \quad (12-c)$$

substituting equations (12-a) and (12-b) into equation (9), and after rearranging we get

$$u_0^2 - 4\sigma_p (\cos \phi_z') u_0 - 4\sigma_p \sigma_p \cos \phi_z' \leq 0 \quad (13-a)$$

and substituting equations (12-a) and (12-c) into equation (9) we get

$$u_1^2 + 4\sigma_p (\cos \phi_z') u_1 - 4\sigma_p \sigma_p \cos \phi_z' \leq 0 \quad (13-b)$$

Both polynomials in the left hand sides of the inequalities (13-a) and (13-b) have a positive real root and a negative real root, so that these

two conditions will be satisfied if  $u_0$  and  $u_1$  are nonnegative and smaller than or equal to the positive-real root of the left hand side polynomials in (13-a) and (13-b) respectively.

Thus solving for the positive real root and then substituting the result into equations (12-b) and (12-c) gives, for each value of  $\phi_z' \leq 90^\circ$ , the maximum and minimum values of  $\omega_z$ , for which the necessary and sufficient condition for the positive realness of  $Z(s)$  is satisfied.

Alternatively, we substitute equations (12-b) and (12-c) into equation (9) and get

$$u^2 - 4\sigma_z \sigma_p \leq 0$$

and

$$u_1^2 - 4\sigma_z \sigma_p \leq 0 .$$

Solving these two equations for the positive real root and substituting the results into the expression for  $\omega_z$ , gives the maximum and minimum values of  $\omega_z$  for which the positive realness of  $z(s)$  is satisfied. Thus we have in this case the two equations

$$\max \omega_z = \omega_p + 2 \sqrt{\sigma_z \sigma_p} \quad (14-a)$$

and

$$\min \omega_z = \omega_p - 2 \sqrt{\sigma_z \sigma_p} . \quad (14-b)$$

Note that these equations will give the same results as equations (13-a) and (13-b) when poles of  $Z(s)$  are complex, and will also give the realizable area of zeros of  $Z(s)$  when the poles are real.

Thus given  $Z(s)$  with complex conjugate poles in the left half plane, or with negative real poles,  $Z(s)$  will be realizable if the zeros of  $Z(s)$  are complex and lie on or within the curves of equations (14-a) and (14-b).

In the special case where  $Z(s)$  has real poles and complex zeros, it has been shown that  $z(s)$  can be realized by a five element series parallel circuit [11].

The key restriction in this case is given by

$$p'' (p'' - p') - (z' - p'')(z'' - p'') \geq 0 \quad (15)$$

where  $p'$ ,  $p''$ ,  $z'$  and  $z''$  are poles and zeros of

$$Z(s) = \frac{(s + z')(s + z'')}{(s + p')(s + p'')}$$

and  $p'' > p'$ .

If we substitute  $\sigma$ 's and  $\omega$ 's for the  $p$ 's and  $z$ 's of (15), then we get

$$p' = -\sigma_p + \sqrt{\sigma_p^2 - \omega_p^2}$$

$$p'' = -\sigma_p - \sqrt{\sigma_p^2 - \omega_p^2}$$

$$z' = -\sigma_z + \sqrt{\sigma_z^2 - \omega_z^2}$$

$$z'' = -\sigma_z - \sqrt{\sigma_z^2 - \omega_z^2}$$

we also obtain

$$2\sigma_z \sigma_p + 2\sigma_z \sqrt{\sigma_p^2 - \omega_p^2} - \omega_z^2 - \omega_p^2 \geq 0$$

rearranging terms, we have

$$\omega_z^2 + \omega_p^2 \leq 2\sigma_z \sigma_p + 2\sigma_z \sqrt{\sigma_p^2 - \omega_p^2}.$$

Now since

$$\omega_z^2 + \omega_p^2 = (\omega_z - \omega_p)^2 + 2\omega_z \omega_p,$$

then we get

$$(\omega_z - \omega_p)^2 \geq 2\sigma_z \sigma_p + 2\sigma_z \sqrt{\sigma_p^2 - \omega_p^2} - 2\omega_z \omega_p.$$

Comparing this condition to that for the general case in equation (9), we obtain

$$2\sigma_z \sigma_p + 2\sigma_z \sqrt{\sigma_p^2 - \omega_p^2} < 4\sigma_z \sigma_p,$$

and since  $\sigma_z$ ,  $\sigma_p$ ,  $\omega_z$  and  $\omega_p$  are all positive numbers, the result becomes

$$(\omega_z - \omega_p)^2 \leq 2\sigma_z \sigma_p + 2\sigma_z \sqrt{\sigma_p^2 - \omega_p^2} - 2\omega_z \omega_p < 4\sigma_z \sigma_p$$

so that this is indeed more restrictive than the general case and we should not be surprised to find the area of realizability for this case enclosed by that of the general case.

In fact this argument could be used as a proof of the realizability of  $Z(s)$  given in (15), since the area of this case is always enclosed by that of the general case as shown above.

The construction of the realizability region in this case will be in accordance with reference [11], that is

$$p'' M = r = p''^{\frac{1}{2}} (p'' - p')^{\frac{1}{2}},$$

where  $p'' > p'$ .

### C. SAMPLE ILLUSTRATIONS

(a) Complex poles ( $\omega_p > \sigma_p$ )

Let  $\sigma_p = 3$

and  $\omega_p = 5$ .

Then from equations (14-a) and (14-b) we get

$$\begin{aligned}\omega_z \text{ max} &= \omega_p + 2\sqrt{\sigma_z \sigma_p} \\ &= 5 + 2\sqrt{3\sigma_z} = 5 + \sqrt{12\sigma_z},\end{aligned}$$

and

$$\begin{aligned}\omega_z \text{ min} &= \omega_p - 2\sqrt{\sigma_z \sigma_p} \\ &= 5 - \sqrt{12\sigma_z}.\end{aligned}$$

We obtain the limitation of  $\sigma_z$  as follows:

$$\sigma_z < \omega_z \text{ for complex zeros.}$$

If we let  $\omega_z \text{ max} = \sigma_z$ , then we get

$$\begin{aligned}\omega_z &= 5 + \sqrt{12\sigma_z} \\ &= 20.8 = \sigma_z, \text{ maximum value for } \sigma_z, \text{ for } \omega_z \text{ max.}\end{aligned}$$

Similarly if  $\omega_z \text{ min} = \sigma_z$ , then

$$\omega_z = 1.2 = \sigma_z, \text{ maximum value for } \sigma_z, \text{ for } \omega_z \text{ min.}$$

#### (b) Real Poles

Let  $\sigma_p = 2.5$  and

$$\omega_p = 2 \quad \text{with poles at } (-1, -4).$$

Then we obtain

$$\begin{aligned}\omega_z \text{ max} &= \omega_p + 2\sqrt{\sigma_z \sigma_p} \\ &= 2 + \sqrt{10\sigma_z}\end{aligned}$$

and

$$\omega_z \text{ min} = \omega_p - \sqrt{10\sigma_z} \text{ .}$$

If  $\omega_z \text{ max} = \sigma_z$ , then

$$\omega_z = 2 + \sqrt{10\sigma_z}$$

$$= 13.7 = \sigma_z, \text{ max } \sigma_z \text{ for } \omega_z \text{ max and complex zeros.}$$

From the examples given in this section it can be seen that the realizability region of a biquadratic function is dependent upon the restriction we place on the function. That is, as we place more restrictions on the function and proceed from a general function with complex poles to one with real poles which is realizable by certain technique, we see that at each step the realizability region decreases. This decrease of freedom from the additional restriction is to be expected.

### III. COMPUTER STUDY ON REALIZABILITY

For a given location of the complex pole pair ( $\sigma_p$  and  $\omega_p$  specified) in the open left half plane,  $C_0$  and  $C_1$  are simply plots of  $\omega_z$  versus  $\phi_z'$ . Thus the computer program was written to accept  $\sigma_p$  and  $\omega_p$  as inputs and to generate  $C_0$  and  $C_1$  on either the line printer or the Calcomp plotter. Some sample outputs are shown in Figure A-1.A and A-1.B in Appendix A.

The curves are for a normlized set of pole locations with

$$\omega_p = 1$$

and

$$0^\circ < \phi_p' < 90^\circ,$$

where  $\phi_p'$  is the compliment of the argument of the pole  $p$ .

Varying  $\phi_p'$  from  $0^\circ$  to  $90^\circ$  corresponds to shifting the poles along a circular path from the negative real axis to the  $j\omega$  axis.

The program was then modified to produce a set of curves  $C_0$  and  $C_1$  for various pole locations. Given a specific pair of poles, two types of plots may be obtained, depending upon the requirements of the problem.

If an accurate plot of the realizability region is required, the program can be used so that a plot of the realizability region is required, the program can be used so that a plot similar to that shown in Figure A-1.A is obtained. Otherwise if a quick estimate is all that is desired, a plot similar to the one in Figure A-1.B can be used.

Other outputs were obtained for various pole locations.

Results obtained for the following cases are included, with program listing, in Appendix A and B respectively:

1. Complex poles with  $\sigma_p$  moves toward a constant  $\omega_p$ .

2. Complex poles with  $\omega_p$  moves toward a constant  $\sigma_p$ .
3. Complex poles that are moving toward the origin.
4. Real negative poles with one of which moving toward the origin while the other remains stationary.
5. Real negative poles with one of which moving toward infinity while the other remains stationary.
6. Double real poles moving from the neighborhood of the origin toward infinity.
7. Pure imaginary poles moving toward the origin.
8. Pure imaginary poles moving away from the origin.

The above cases are now described in the following sections. In each case a figure is included showing the pole locations with their values shown in the accompanying table, in which the location of the corresponding computer output plots are indicated.

.

### Case 1

This case illustrates the zeros region for given complex poles with

$\sigma_p$  moves toward a constant  $\omega_p$ .

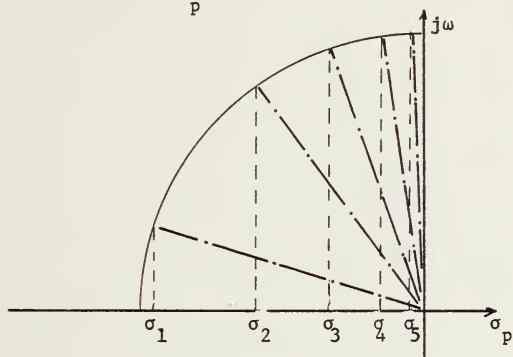


Figure 2.

Five different poles ( conjugate poles are not shown ) are given in accordance to show the zero regions for each  $\sigma_p$  as  $\omega_p$  remains constant. The locations of each pole are tabulated in Table 1.

Table 1

Pole	$\sigma_p$	$\omega_p$	Computer Plot
$C_1$	0.999	1.0	Figure A-1.A,B
$C_2$	0.707	1.0	Figure A-2
$C_3$	0.55	1.0	Figure A-3
$C_4$	0.25	1.0	Figure A-4
$C_5$	0.1	1.0	Figure A-5

## Case 2

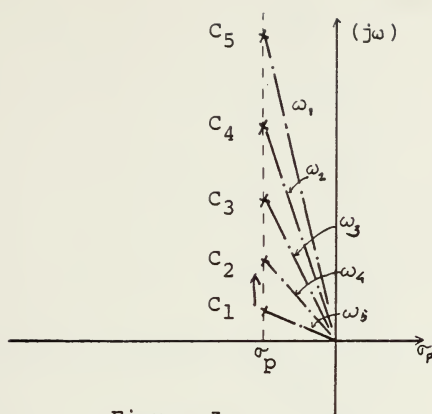


Figure 3.

This case illustrates the zeros region for given complex poles with

$\omega_p$  moves toward a constant  $\sigma_p$ .

Five different pole locations are shown in Table 2.

Table 2

Pole	$\sigma_p$	$\omega_p$	Computer Plot
$C_1$	1.0	1.05	All plots are shown in figure A-7.
$C_2$	1.0	2.0	
$C_3$	1.0	3.0	
$C_4$	1.0	5.0	
$C_5$	1.0	8.0	

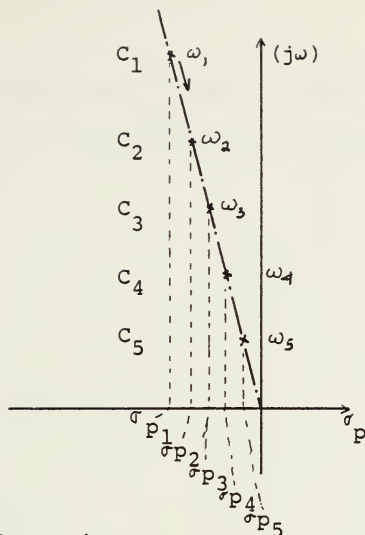
Case 3

Figure 4.

This case illustrates the zeros region for given complex poles that are moving toward the origin. Five different pole locations are shown in Table 3.

Table 3

Pole	$\sigma_p$	$\omega_p$	Computer Plot
$C_1$	4.0	30.652	All plots are shown in Figure A-8.
$C_2$	2.0	15.326	
$C_3$	0.5	3.834	
$C_4$	0.1	0.766	
$C_5$	0.01	0.077	

#### Case 4

This case illustrates the zeros region for given real negative poles with one of which moving toward the origin while the other remains stationary.

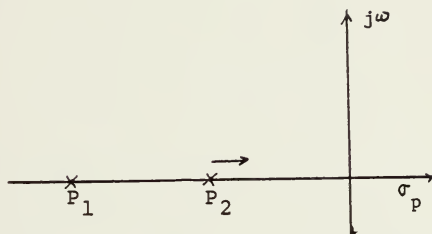


Figure 5.

Five different pole locations are shown in Table 4.

Table 4

Condition	Pole $P_1$	Pole $P_2$	$\sigma_p$	$\omega_p$
$C_1$	8.0, 0	4.0, 0	6.0	5.657
$C_2$	8.0, 0	1.0, 0	4.5	2.828
$C_3$	8.0, 0	0.5, 0	4.25	2.0
$C_4$	8.0, 0	0.1, 0	4.05	1.414
$C_5$	8.0, 0	0.05, 0	4.025	0.632

All plots are shown in computer output Figure A-9.

### Case 5

This case illustrates the zeros region for given real negative poles with one of which moving toward infinity while the other remains stationary.

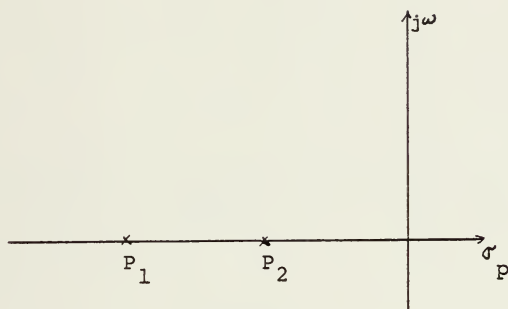


Figure 6.

Five different pole locations are shown in Table 5.

Table 5

Condition	Pole $P_1$	Pole $P_2$	$\sigma_p$	$\omega_p$
$C_1$	8.0,0	2.0, 0	5.0	4.0
$C_2$	15.0,0	2.0, 0	8.5	5.477
$C_3$	30.0,0	2.0, 0	16.0	7.746
$C_4$	50.0,0	2.0, 0	26.0	10.0
$C_5$	100.0,0	2.0, 0	56.0	14.142

All plots are shown in computer output Figure A-10.

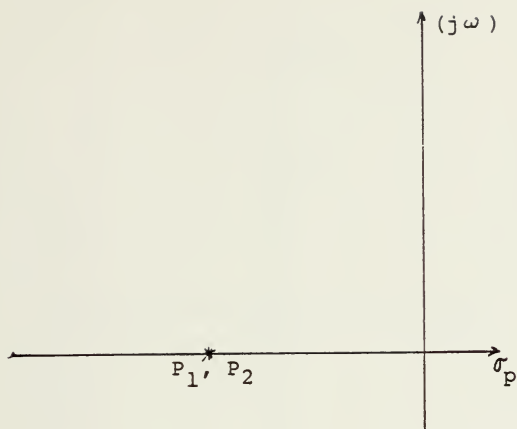
Case 6

Figure 7.

This case illustrates the zeros region for given double real poles moving from the neighbourhood of the origin toward infinity. Five different pole locations are shown in Table 6.

Table 6

Condition	Pole $P_1=P_2$	$\zeta_p$	$\omega_p$	Computer Plot
$C_1$	1.0, 0	1.0	1.0	All plots are shown in computer output
$C_2$	2.0, 0	2.0	22.0	
$C_3$	5.0, 0	5.0	5.0	
$C_4$	10.0, 0	10.0	10.0	
$C_5$	25.0, 0	25.0	25.0	

Case 7

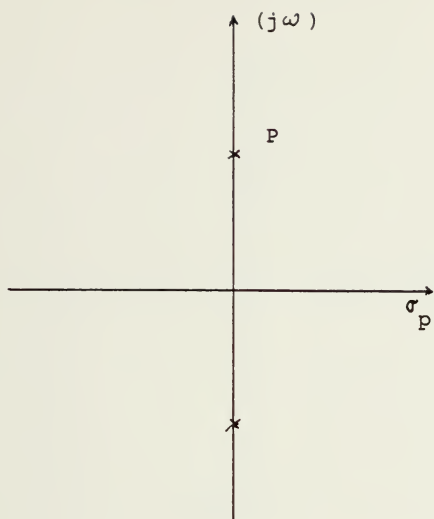


Figure 8.

This case illustrates the zeros region for given pure imaginary poles moving toward the origin. Five different pole locations are shown in Table 7.

Table 7

Condition	$\sigma_p$	$\omega_p$	Computer Plot
$C_1$	0.0	3.0	All plots are shown in Figure A-12.
$C_2$	0.0	1.0	
$C_3$	0.0	0.5	
$C_4$	0.0	0.1	
$C_5$	0.0	0.05	

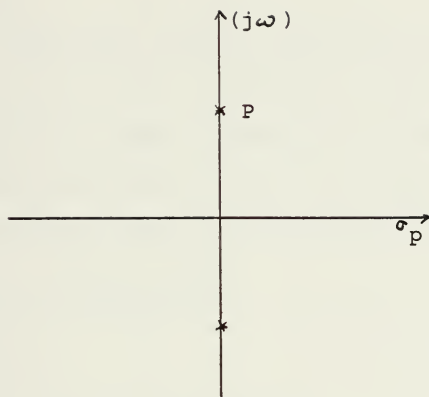
Case 8

Figure 9.

This case illustrates the zeros region for given pure imaginary poles moving away from the origin. Five different pole locations are shown in Table 8.

Table 8

Condition	$\sigma_p$	$\omega_p$	Computer Plot
$C_1$	0.0	4.0	All plots are shown in computer output Figure A-13.
$C_2$	0.0	10.0	
$C_3$	0.0	20.0	
$C_4$	0.0	100.0	
$C_5$	0.0	200.0	

#### IV. SENSITIVITY STUDY

##### A. INTRODUCTION

It has been shown [12], that in a linear and not necessarily reciprocal network, with only one independent source and initially at rest, the ratio of the Laplace transform of the current in any branch or of the voltage across any node pair to the Laplace transform of either an independent voltage or current source is given by

$$T(x) = \frac{W T_x(0) + x T_x(\infty)}{W + x} \quad (16)$$

where

$x$  = an adjustable immitance.

$T(x)$  = the network function relating a response to an  
excitation.

$T_x(0) = \lim_{x \rightarrow 0} T(x)$  = the network function evaluated with  
 $x = 0$ ,  $T_x(0)$  is assumed to be finite.

$T_x(\infty) = \lim_{x \rightarrow \infty} T(x)$  = the network function evaluated when  
 $x = \infty$ ,  $T_x(\infty)$  is assumed to be finite.

$W$  = the Thevenin immitance (independent of  $x$ ) seen looking  
back into network from the terminals of the adjustable  
immitance.  $W$  is assumed to be finite.

Equation (16) is generic in form in that  $x$  can be either an impedance or admittance. In this paper the symbol  $T(x)$  is used to denote a driving point immitance.

## B. GRAPHICAL REPRESENTATION OF $T(x)$

Consider equation (16) for the case of a driving point immittance function as shown in Figure 10.

Dividing numerator and denominator of equation (16) by  $W$ , we get

$$T(x) = \frac{T_x(0) + \frac{x}{W} T_x(\infty)}{1 + \frac{x}{W}}$$

which can be rewritten as,

$$T(x) = \frac{T_x(\infty) \left(1 + \frac{x}{W}\right) + T_x(0) - T_x(\infty)}{1 + \frac{x}{W}}$$

or

$$T(x) = T_x(\infty) + \frac{T_x(0) - T_x(\infty)}{1 + \frac{x}{W}} \quad (17)$$

Let

$$T(x) = T_x(\infty) + V \quad (18)$$

where

$$V = \frac{T_x(0) - T_x(\infty)}{1 + \frac{x}{W}} \quad (19)$$

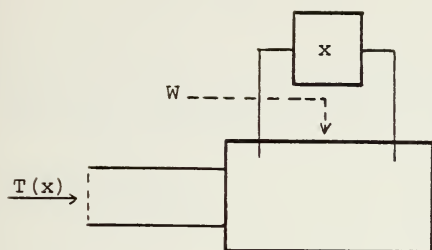


Figure 10.

Driving point immittance with variable component  $x$ .

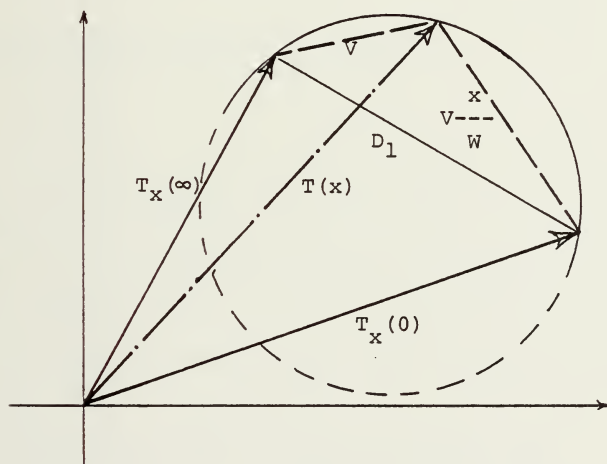


Figure 11.

Graphical representation of  $T(x)$  phasor.

From equation (19) we have

$$V + V \frac{x}{W} = T_x(0) - T_x(\infty) \quad (20)$$

we note that  $T_x(0)$  and  $T_x(\infty)$  are fixed quantities.

Now treat each quantity in (20) as phasor, i.e.

$$V = |V| \exp. j\theta_1$$

and

$$W = |W| \exp. j\theta_2$$

then the left hand side of equation (20) represents the sum of two phasors.

If  $x$  is real, then the phasor

$$V \frac{x}{W} = x \left| \frac{V}{W} \right| \exp. j(\theta_1 - \theta_2)$$

is lagging behind the phasor  $V$  by an angle

$$\theta = \theta_1 - \theta_2.$$

Since  $T_x(0)$  and  $T_x(\infty)$  are two fixed quantities, the right hand side of equation (20) is constant so that the left hand side of (20) is constant. This requires that the angle must be constant (as  $x$  varies).

This implies that the tip of phasor  $V$  describes the arc of the circle, of which the quantity  $T_x(0) - T_x(\infty)$  is a chord as shown in Figure 11.

To construct the circle we proceed as follows [13]:

1. Plot  $T_x(\infty)$  and  $T_x(0)$  as two phasors from the origin.
2. Draw a straight line  $D_1$  through the tips of the two phasors in step 1.

3. Calculate the angle and draw a straight line  $D_2$  from the tip of  $T_x(0)$  forming angle with  $D_1$  having the value  $-\theta$  measuring clockwise from  $D_1$ .

Therefore  $D_2$  is tangent to the circular locus of  $T_x$ .

The centre of this circle,  $O'$ , is located at the intersection of the two perpendicular lines erected on the chord in the middle of  $T_x(0) - T_x(\infty)$  and  $D_1$  at the tip of  $T_x(0)$ , as shown in Figure 12.

Knowing the locus of  $T(x)$  as  $x$  varies from zero to infinity, we can graphically determine  $T(x)$  at any value of  $x$ .

### C. GRAPHICAL REPRESENTATION OF SENSITIVITY PHASOR

If we define the sensitivity as

$$S_x^{T(x)} = \frac{d T(x)}{dx}$$

then with equation (16) we can obtain, [13]

$$S_x^{T(x)} = \frac{T_x(0) - T_x(\infty)}{W \left[ 1 + \frac{x}{W} \right]^2}$$

or

$$\frac{x S_x^{T(x)}}{V \frac{x}{W}} = - \frac{V}{T_x(0) - T_x(\infty)}$$

where

$$V = \frac{T_x(0) - T_x(\infty)}{1 + \frac{x}{W}} .$$

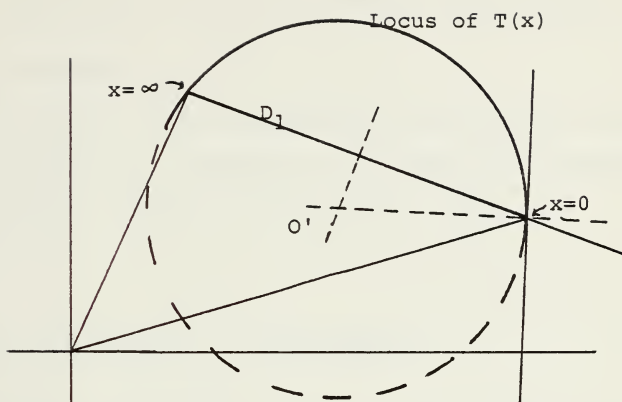


Figure 12.

Graphical representation of locus of  $T(x)$ .

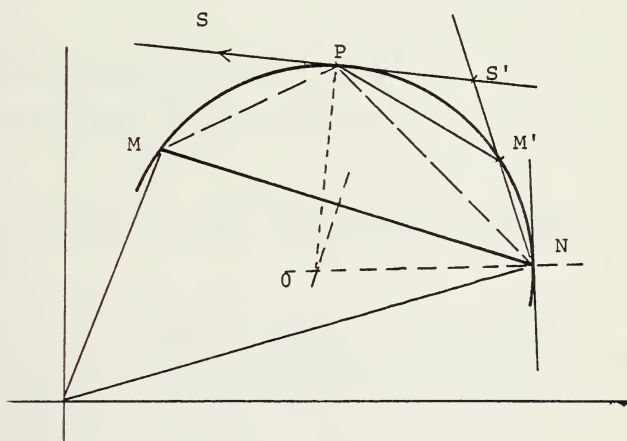


Figure 13.

Graphical representation of sensitivity phasor  $S$ .

It can be shown [13] that the sensitivity of a function may be graphically represented by a phasor such as  $\overline{PS}$  as shown in Figure 13.

#### D. SAMPLE ILLUSTRATION

As an illustration of the above discussion in this chapter, let us consider a biquadratic p.r. function as an RC driving point admittance, which can be realized in Foster 2 form as shown in Figure 14.

Let

$$Y(s) = \frac{s^2 + 4s + 3}{s^2 + 6s + 8} = \frac{(s+1)(s+3)}{(s+2)(s+4)}$$

which can be realized in Foster 2 form with the element value as given in Figure 5, in which all resistance and capacitances are measured in Ohms and Farads respectively.

Now let  $R_2$  change by the amount  $\Delta R$ , such that

$$0 < \Delta R < \infty .$$

Thus, Figure 15 is modified as shown in Figure 16.

We can calculate  $T_x(0)$ ,  $T_x(\infty)$  and  $W$  as follows:

$$\begin{aligned} T_x(0) &= Y(s, \Delta R=0) = \frac{1}{R_0} + \frac{1}{R_1 + 1/sC_1} + \frac{1}{R_2 + 1/sC_2} \\ &= .375 + \frac{.125 s}{.5 s + 1} + \frac{.09375 s}{.25 s + 1} = \frac{s^2 + 4s + 3}{s^2 + 6s + 8} . \quad (24) \end{aligned}$$

$$\begin{aligned} T_x(\infty) &= Y(s, \Delta R=\infty) = \frac{1}{R_0} + \frac{1}{R_1 + 1/sC_1} \\ &= .375 + \frac{.125 s}{.5 s + 1} = \frac{2.5 s + 3}{4 s + 8} . \quad (25) \end{aligned}$$

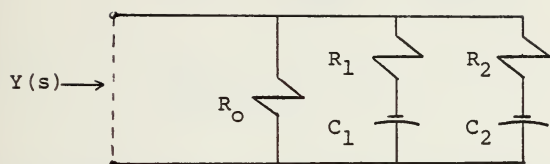


Figure 14.

The realization of Foster-2 form of driving point admittance  $Y(s)$ .

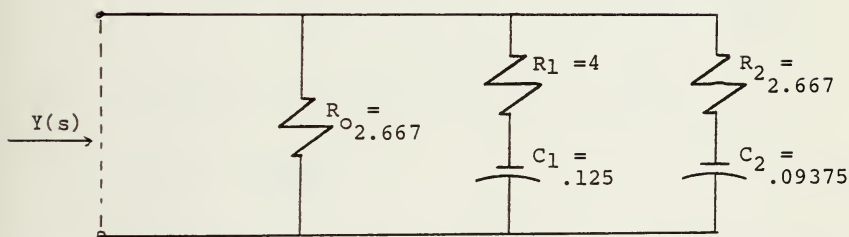


Figure 15.

The realization of biquadratic p.r. driving point admittance in Foster 2 form, where

$$Y(s) = \frac{s^2 + 4s + 3}{s^2 + 6s + 8}.$$

$$\begin{aligned}
W &= R_2 + \frac{1}{sC_2} + \frac{1}{Y(s, \Delta R=\infty)} \\
&= 2.667 + \frac{10.667}{2} + \frac{4s+8}{2.5s+3} \\
&= \frac{14.5s^2 + 40s + 99}{7.5s^2 + 9s} \quad . \quad (26)
\end{aligned}$$

From expression (26) we obtain the expression  $\frac{x}{W}$  to be

$$\frac{x}{W} = \frac{7.5s^2x + 9sx}{14.5s^2 + 40s + 99} \quad . \quad (27)$$

Substitute equations (24), (25), (26) and (27) into equation (22) we get,

$${}_sT_x(x) = - \frac{\frac{s^2 + 4s + 3}{s^2 + 6s + 8} - \frac{2.5s + 3}{4s + 8}}{\frac{14.5s^2 + 40s + 99}{7.5s^2 + 9s} \left[ 1 + \frac{7.5xs^2 + 9xs}{14.5s^2 + 40s + 99} \right]^2} \quad , \quad (28)$$

then for simplification we write equation (28) above as,

$${}_sT_x(x) = - \frac{N(s,x)}{D(s,x)} \quad (29)$$

where  $N(s,x)$  and  $D(s,x)$  represent the numerator and denominator of the equation (28) respectively.

In the frequency domain, the expression in equation (29) can be written as,

$${}_sT_x(j\omega, x) = - \frac{N(j\omega, x)}{D(j\omega, x)} \quad (30)$$

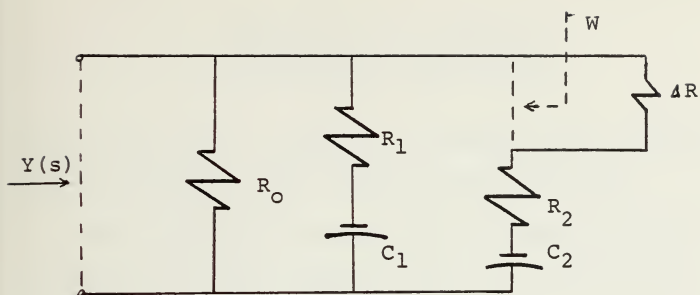


Figure 16.

Driving point admittance  $Y(s)$  with variable resistance  $\Delta R$ .

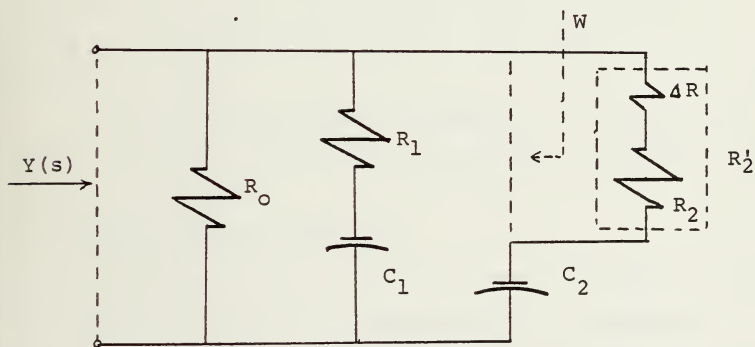


Figure 17.

Driving point admittance  $Y(s)$  with variable resistance  $R_2' = R_2 + \Delta R$ .

$$= - \frac{\operatorname{Re} [N(j\omega, x)] + j \operatorname{Im} [N(j\omega, x)]}{\operatorname{Re} [D(j\omega, x)] + j \operatorname{Im} [D(j\omega, x)]} \quad (30a)$$

where  $\operatorname{Re} [N(j\omega, x)]$ ,  $\operatorname{Re} [D(j\omega, x)]$ ,  $\operatorname{Im} [N(j\omega, x)]$  and  $\operatorname{Im} [D(j\omega, x)]$  are real and imaginary parts of equation (30) respectively.

From equation (30a) we can obtain the magnitude of that sensitivity function. If we write equation (30a) in exponential form we have

$$S_x^{T(x)}(j\omega, x) = |S_x^{T(x)}| \exp(\phi_N - \phi_D) \quad (31)$$

where

$$S_x^{T(x)} = \text{magnitude of equation (30a)}$$

$$= \left[ \frac{\{\operatorname{Re} [N(j\omega, x)]\}^2 + \{\operatorname{Im} [N(j\omega, x)]\}^2}{\{\operatorname{Re} [D(j\omega, x)]\}^2 + \{\operatorname{Im} [D(j\omega, x)]\}^2} \right]^{\frac{1}{2}}.$$

$\phi_N$  = the argument of the numerator of equation (30a).

$\phi_D$  = the argument of the denominator of equation (30a).

#### E. COMPUTER STUDY ON SENSITIVITY

In this section, the sensitivity of the function  $T(x)$  with respect to the variation of one of its components will be investigated by computer. Consider the general expression  $Y(x)$  in equation (16), which is repeated here for convenience.

$$T(x) = \frac{W T_x(s, x=0) + x T_x(s, x=\infty)}{W + s} \quad (16)$$

The sensitivity function of this equation is

$$S_x^{T(x)} = - \frac{T_x(s, x=0) - T_x(s, x=\infty)}{W \left[ 1 + \frac{x}{W} \right]^2} \quad (22)$$

As an example let us consider the function  $Y(s)$ , as we derived in section C,

$$Y(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$$

and the realization of this function in Foster-2 form is shown in Figure 17.

For our investigation, let  $R_2$  change with the amount of  $\Delta R$  in series with  $R_2$  and let

$$R_2' = R_2 + \Delta R.$$

To obtain the sensitivity  $S_{R_2}^{T(s, R_2')}$  in terms of frequency and  $R_2'$ , we use the same calculation from equation (30).

The computer output in Appendix B shows us the plot of the magnitude of sensitivity function (30) versus  $R_2'$ . For this purpose we use equation (31) as reference.

Various cases are investigated using computer, resulting in plots of the magnitude of equation (31) versus  $R_2'$  for different frequencies.

## V. CONCLUSION

In this paper, a comparative study has been made on the realizability conditions of a general biquadratic function. The graphical method of Chan and Phung [10] was used for this study and computer programs are written for verifications. General observations can be made as follows:

1. The realizability for the zeros expands considerably as the poles move from the imaginary axis to the real axis. This results both from  $C_0$  moving away from the origin and  $C_1$  moving toward the origin.

2. The limits on  $C_0$  and  $C_1$  are found easily from equations (3a) and (3b). As  $\sigma_p$  approaches zero, corresponding to the poles moving toward the  $j\omega$  axis, both  $C_0$  and  $C_1$  tend toward the circle  $\omega_z = \omega_p$  for all  $\phi_z'$ . As can be seen from Figure A-6,  $C_0$  and  $C_1$  separate very rapidly for small movements of the poles away from the  $j\omega$  axis. We can conclude that when  $\sigma_p = 0$  we require  $\omega_z = \omega_p$  in order for  $Z(s)$  to be realizable with complex zeros. This can also be seen easily from equation (9).

3. As  $\sigma_p$  approaches  $\omega_p$ , corresponding to the poles moving toward the real axis,  $C_0$  approaches a value of  $-5.828\omega_p$  on the real axis, and  $C_1$  similarly approaches a value of  $-0.172\omega_p$ , as we can see in Figure A-1.A.

A brief study was made on sensitivity functions [13], some computer results are obtained. However, due to time limitation, a complete investigation in sensitivity was not possible and is left as a suggested topic for further studies.

APPENDIX A  
COMPUTER OUTPUT

A. ZERO'S REGION

The illustration of zeros regions will be shown in this appendix.

The computer outputs show us the regions for various given omega and sigma poles.

Various cases are explained as follows:

Figure A-1a and A-1b show the zeros regions for given sigma pole = 0.999 and omega pole = 1.0 using Calcomp and PLOT in the computer program respectively.

All the following plots show us the computer outputs by using Calcomp in the computer program.

Figure A-2, A-3, A-4 and A-5 show us the plots of zeros regions for given sigma poles equal to 0.707, 0.55, 0.25 and 0.1 respectively with the amount of omega pole equal to 1.0 each.

Figure A-6 shows all plots given in Figure A-1a, A-2, A-3, A-4 and A-5 in one computer output.

Figure A-7 shows the plots of zeros regions as pole moves at sigma pole constant.

Figure A-8 shows the zeros regions as pole moves to the origin.

Figure A-9 and A-10 show the plots of zeros regions as pole moves to the origin and to infinity respectively.

Figure A-11 shows the plot of zeros regions of a double real negative poles as both move along the negative axis.

Figure A-12 and A-13 show us the zeros region plots of a given pair of pure negative poles, as both move to the origin and to the infinity respectively.

## B. SENSITIVITY FUNCTION

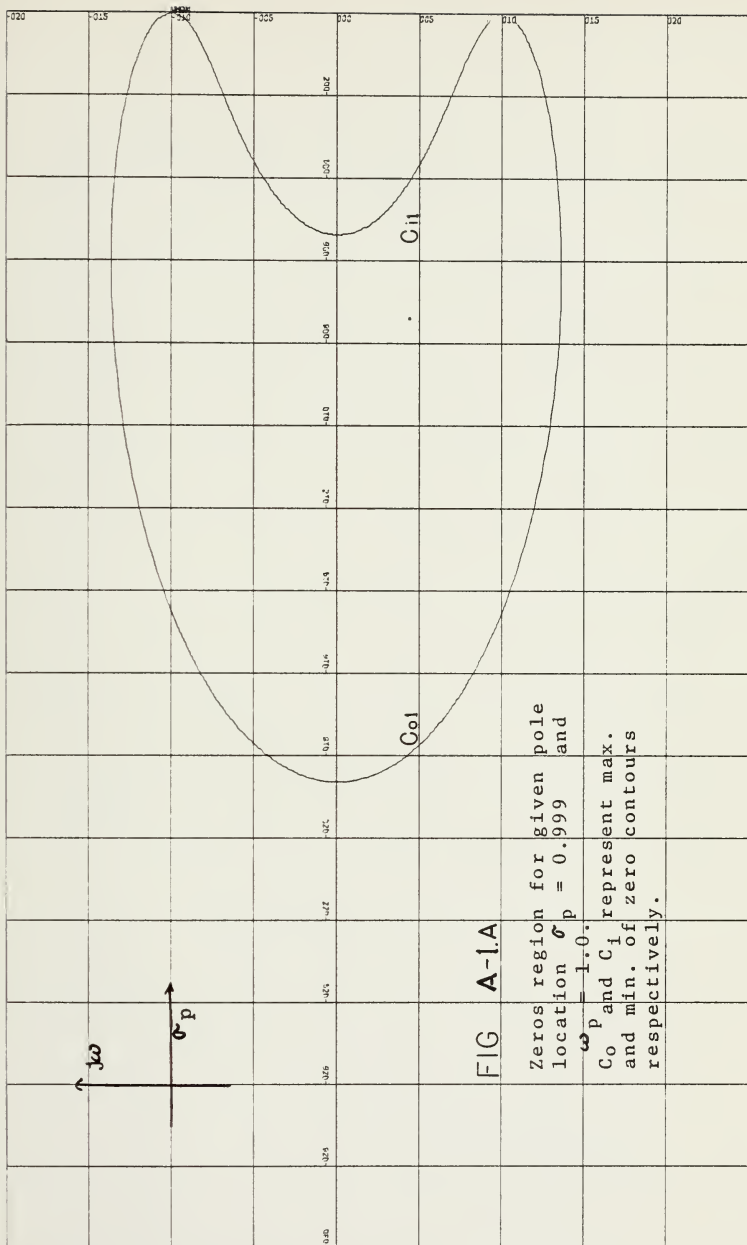
Various cases of sensitivity as a function of  $R'_2$  are illustrated in the computer outputs for different particular frequencies.

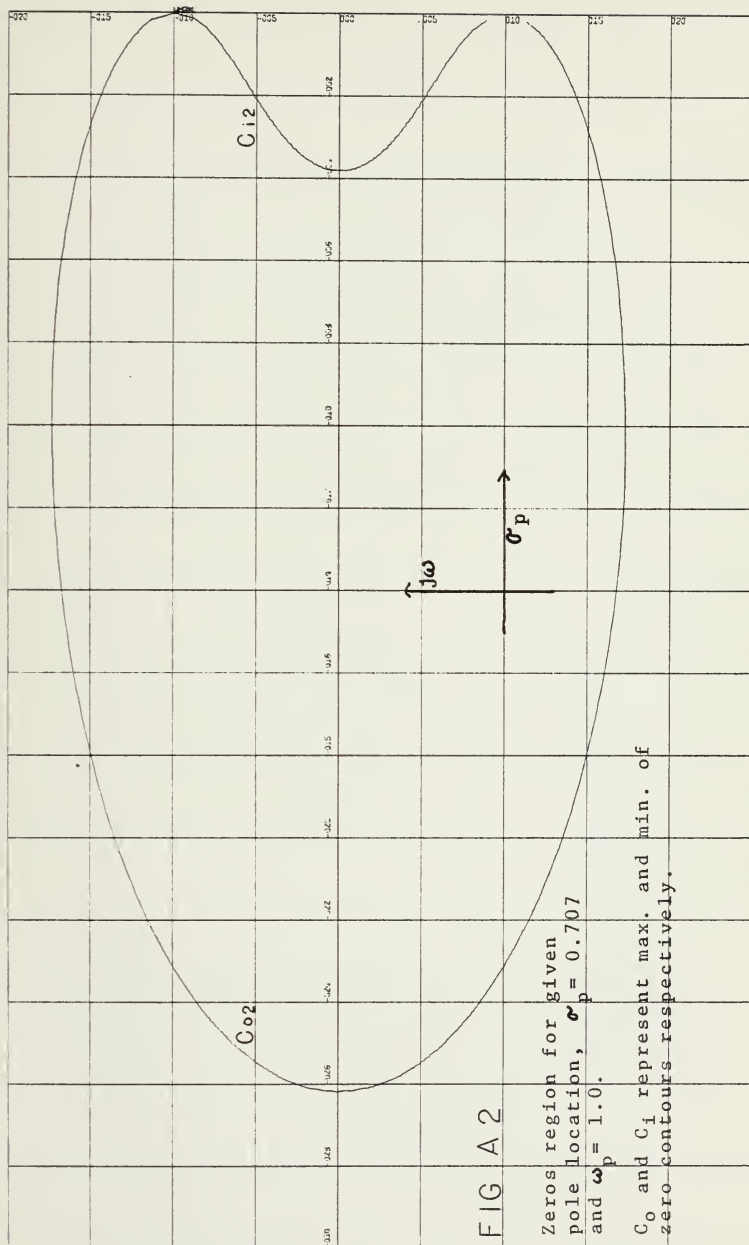
The computer outputs show us the plots of sensitivity versus  $R'_2$  and the tabulations of this plot.

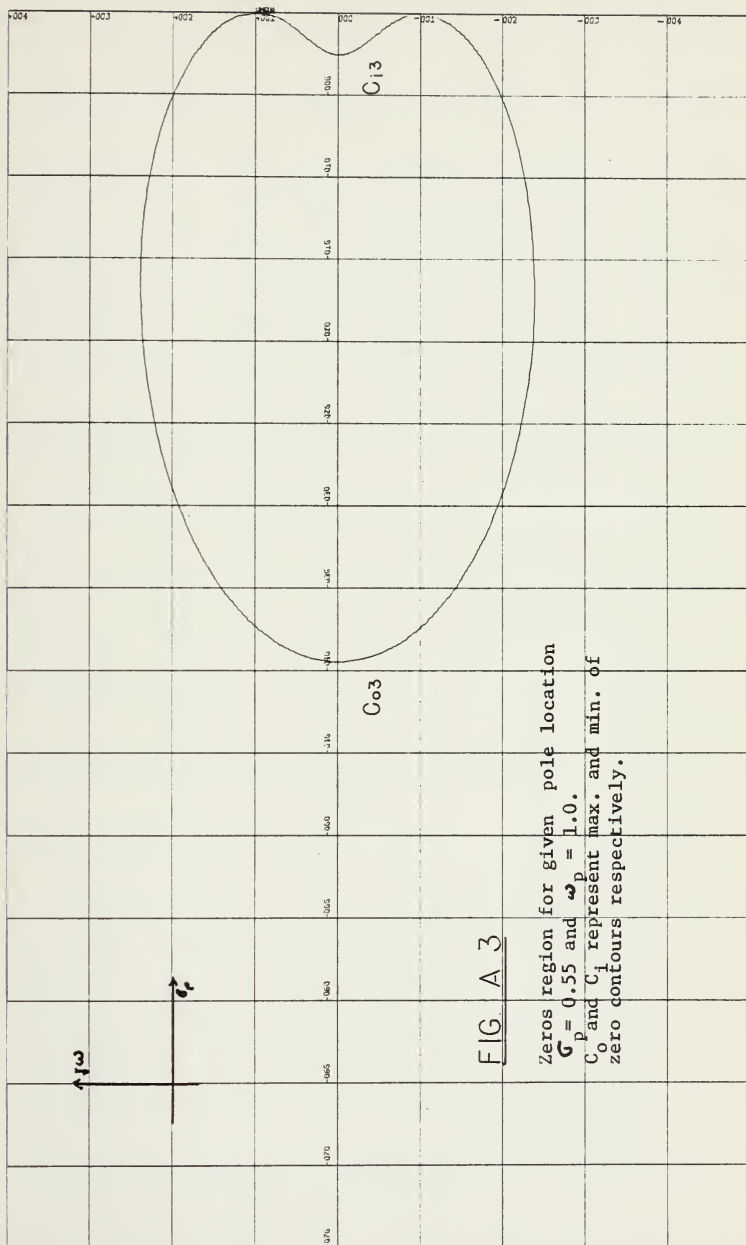
The resistance  $R'_2$  is select to be varied from the amount of 1 Ohm to 50 Ohms with increment to 1 Ohm. And the frequencies are choosen beginning at 1 rad/sec. up to 10,000 rad/sec.

Figure B-1, B-2, ..... B-9 show us the sensitivity functions as  $R'_2$  moves from 1 to 50 Ohms, at the frequencies of 1, 5, 10, 50, 100, 500, 1000, 5000 and 10000 rad/sec. The tabulations of those plots are also shown in Table B-1, B-2 .....B-9 respectively.

Figure B-10 shows us five different plots of sensitivity functions illustrated in one computer output.







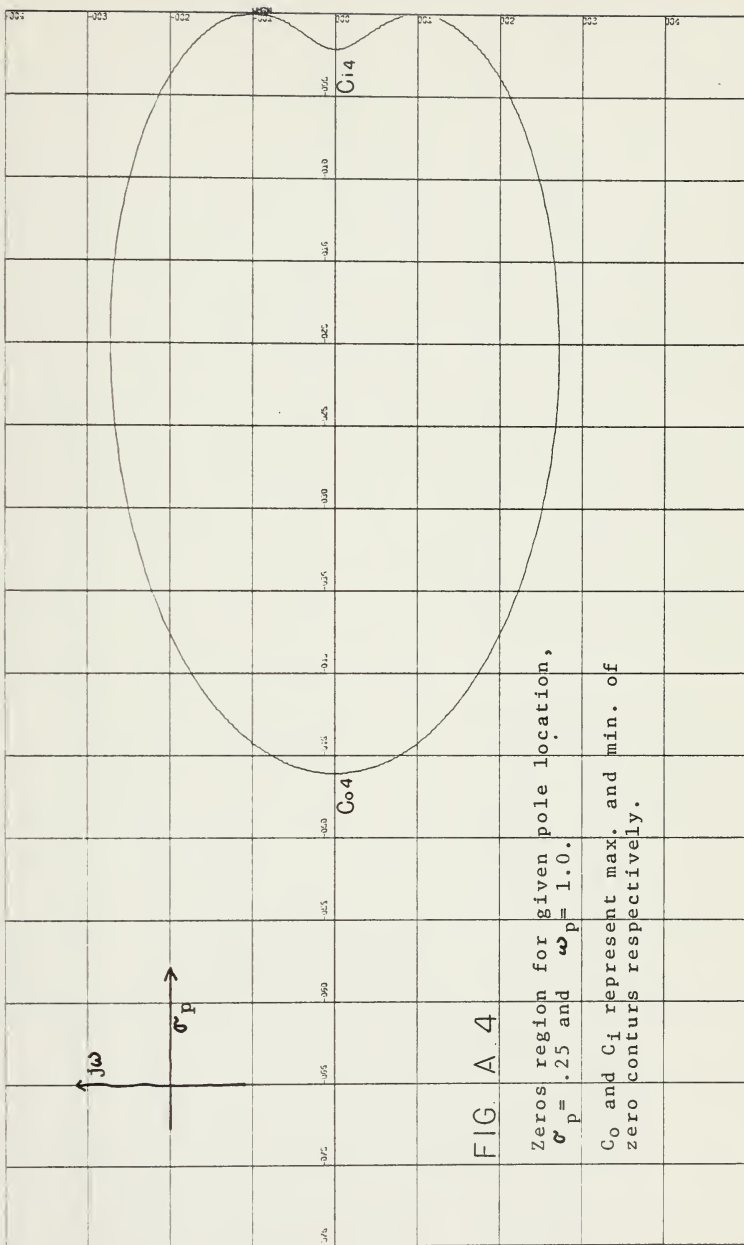
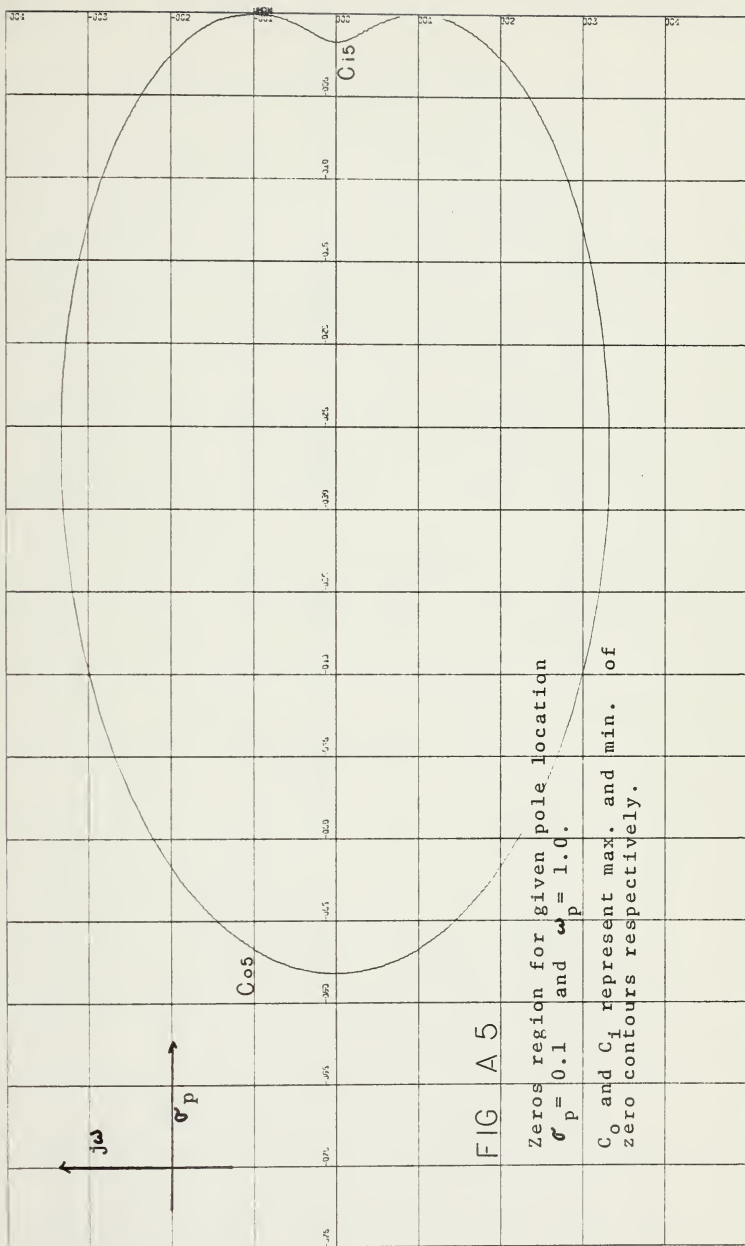


FIG A 4

Zeros region for given pole location,  
 $\sigma_p = .25$  and  $\omega_p = 1.0$ .

$C_o$  and  $C_i$  represent max. and min. of  
 zero contours respectively.



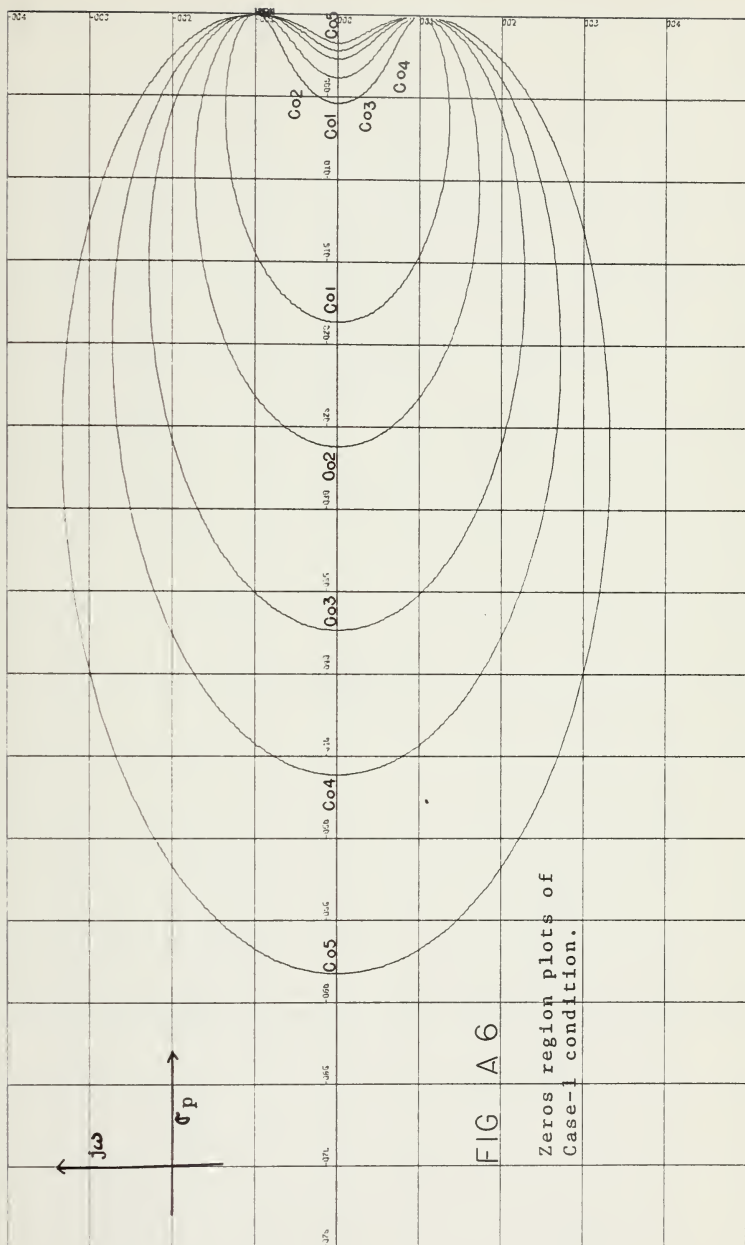


FIG A 6

Zeros region plots of  
Case-1 condition.

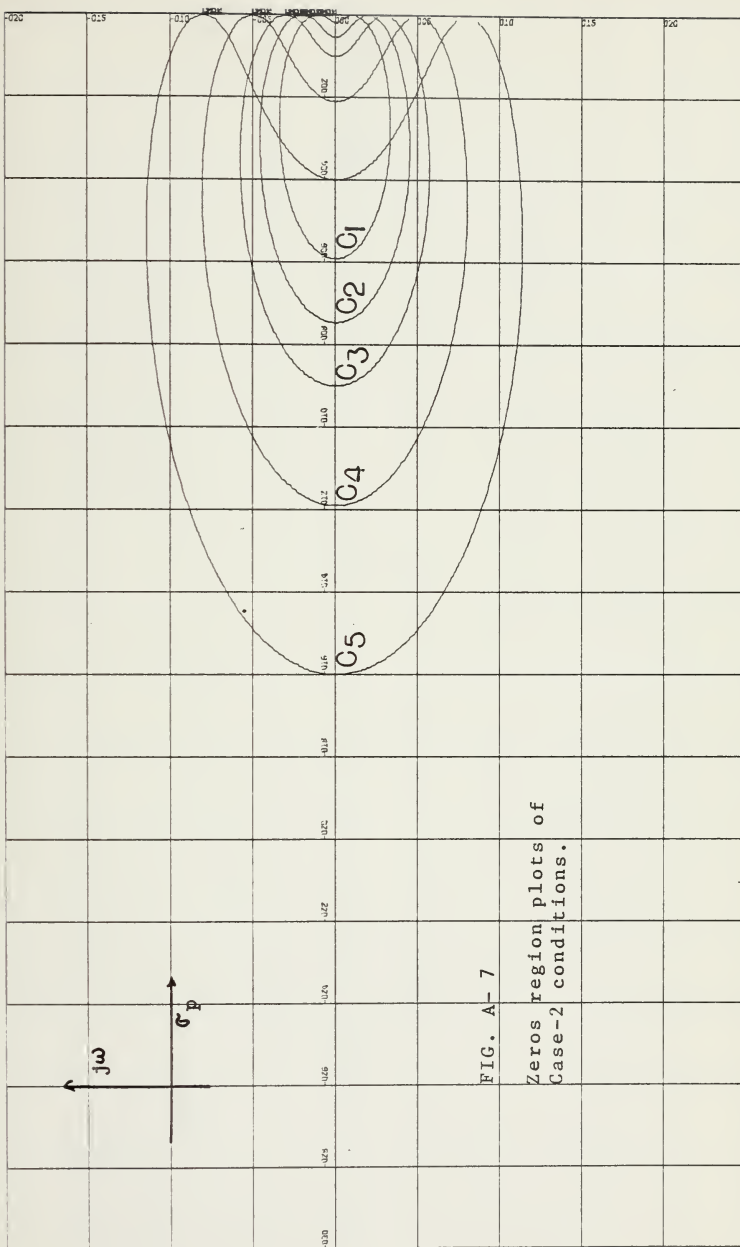


FIG. A-7

Zeros region plots of  
Case-2 conditions.

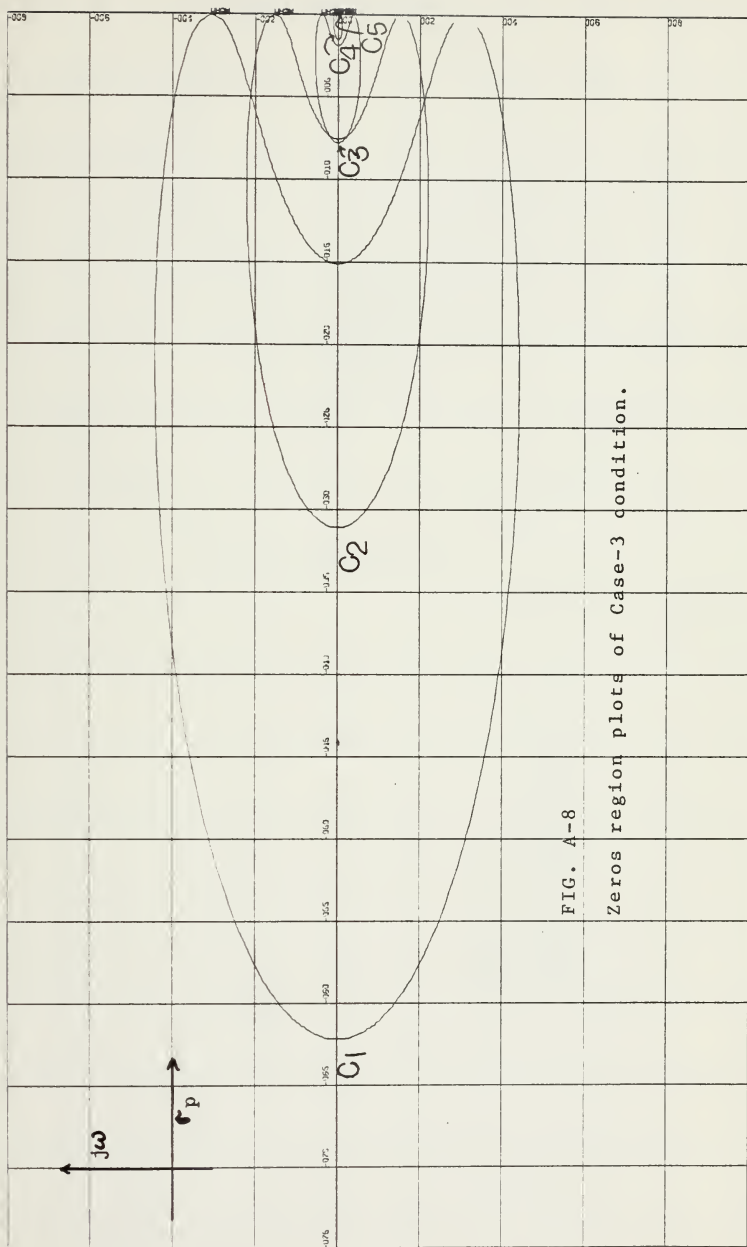
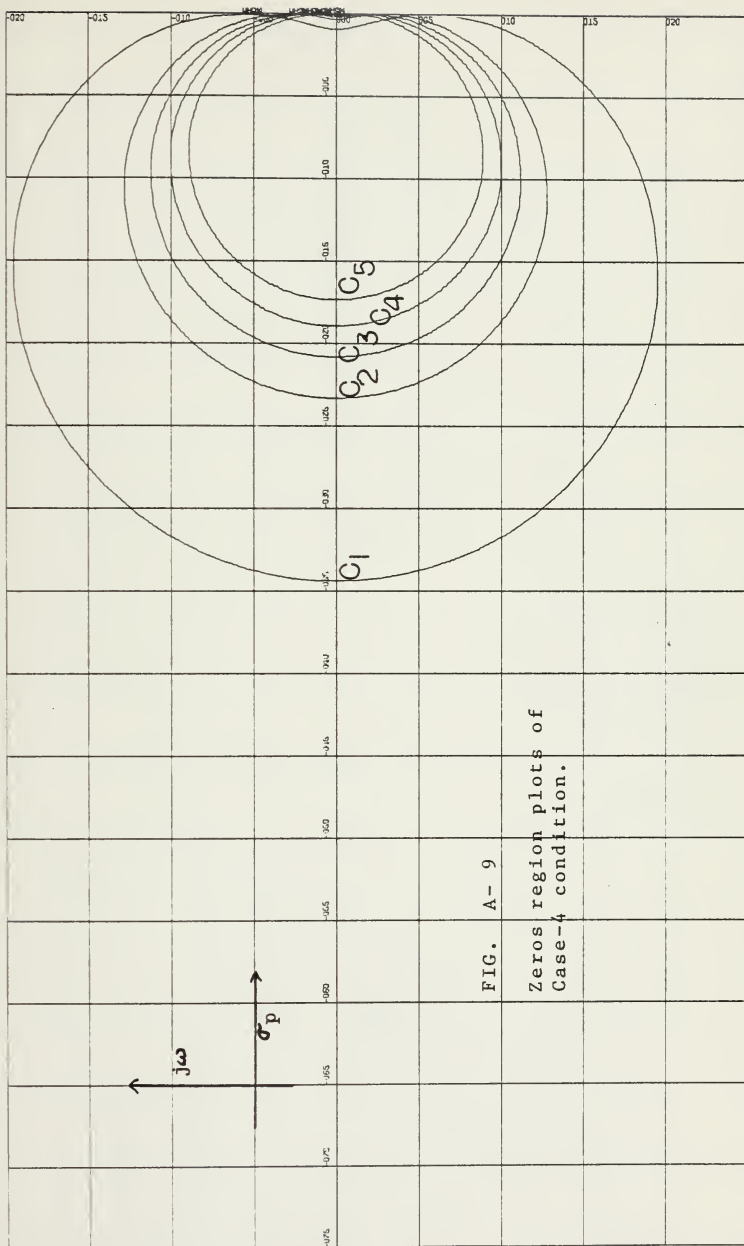


FIG. A-8  
Zeros region plots of Case-3 condition.



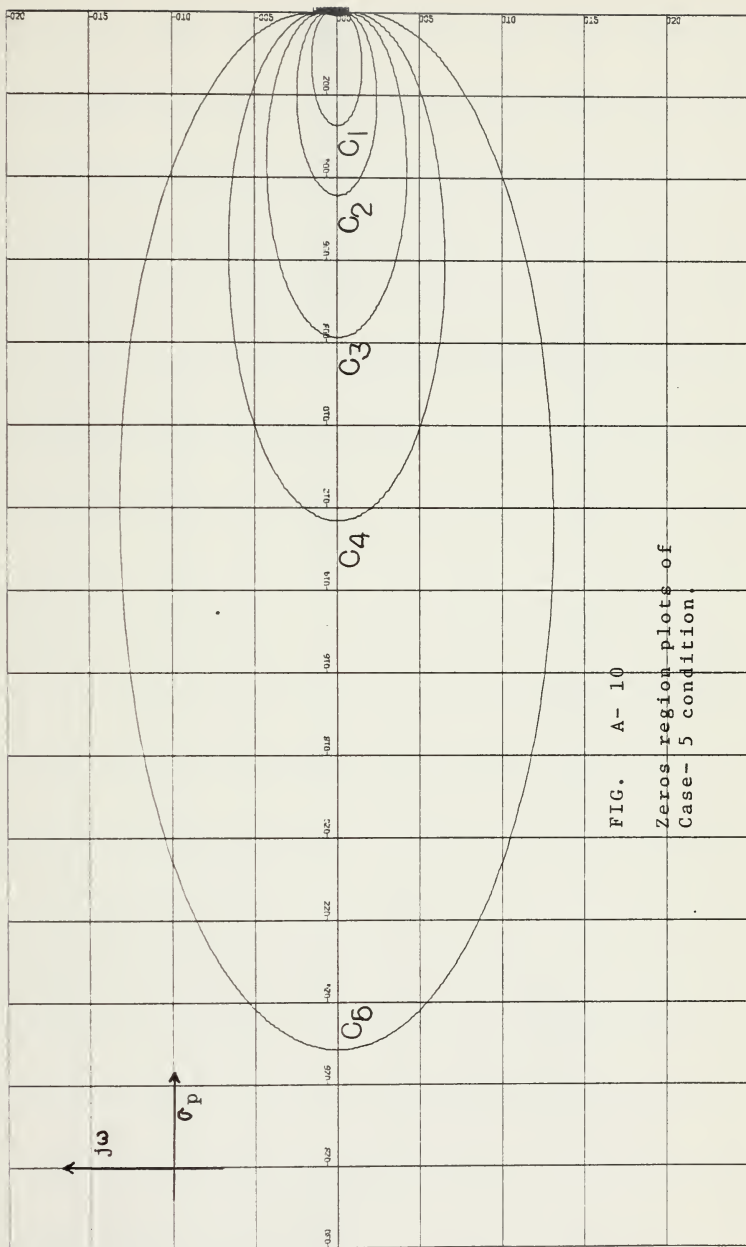


FIG. A-10  
Zeros region plots of  
Case-5 condition.

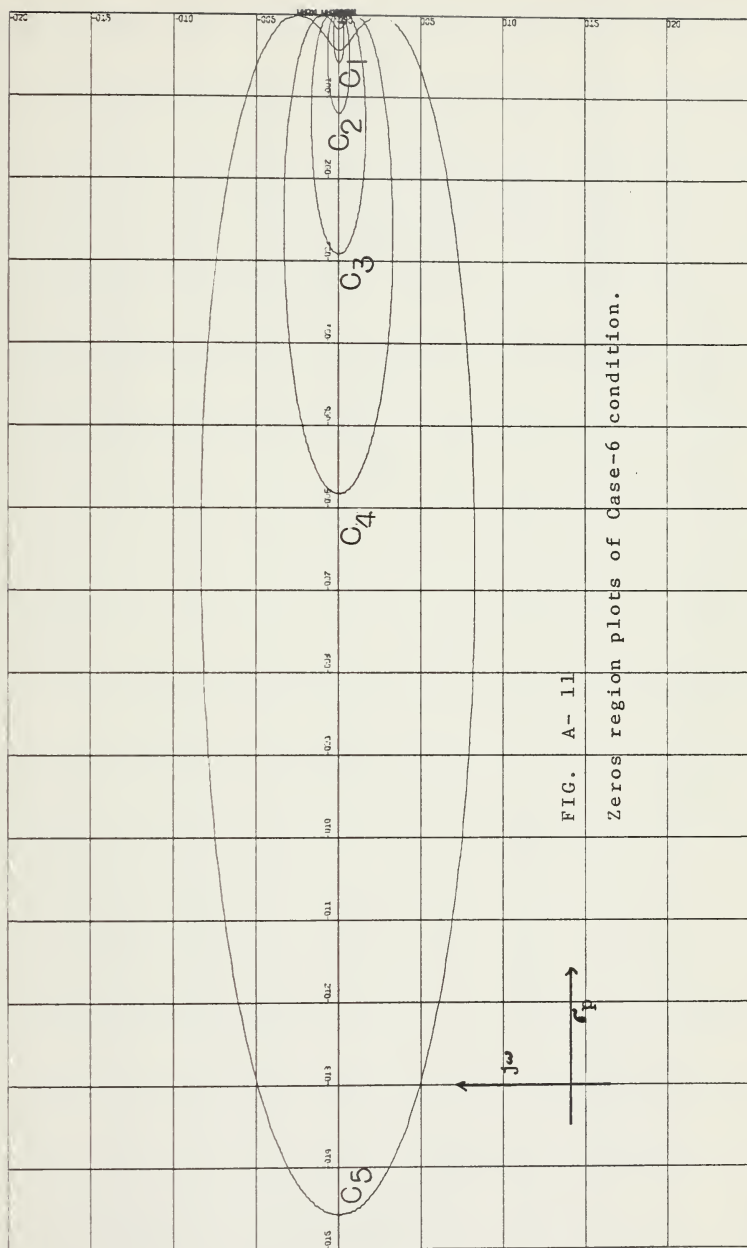


FIG. A-11

Zeros region plots of Case-6 condition.

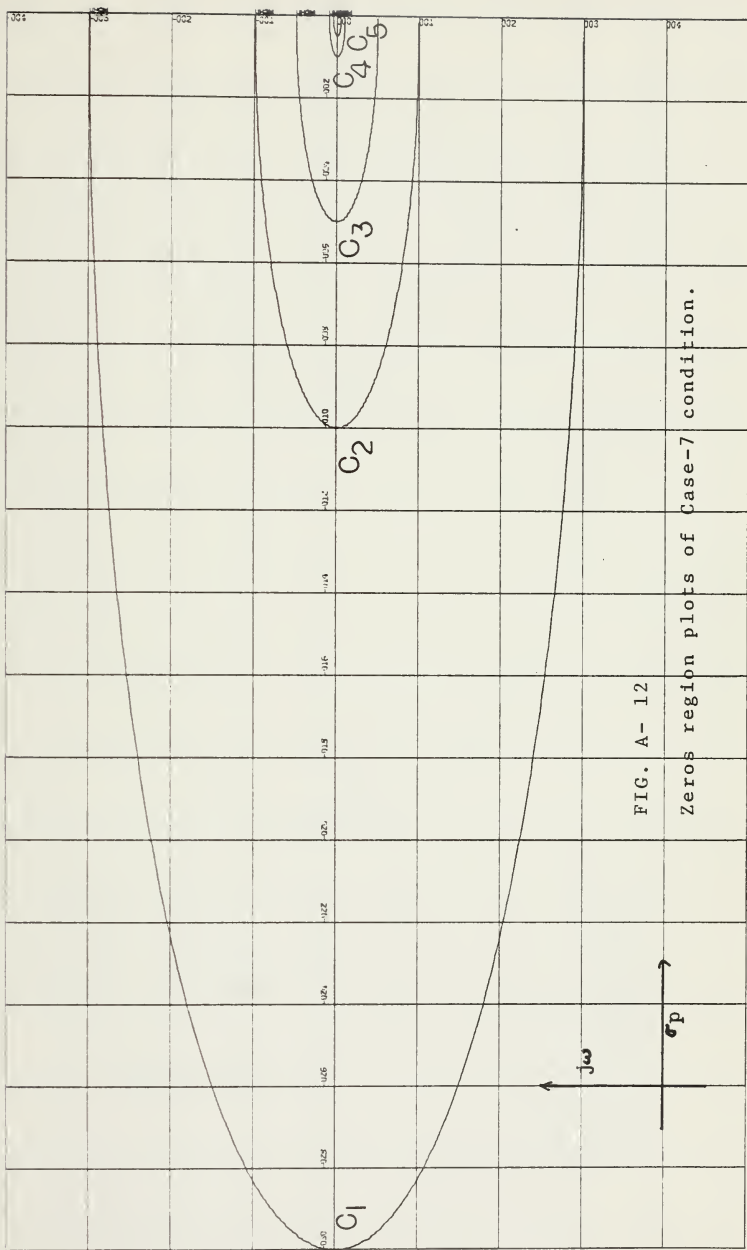


FIG. A-12

Zeros region plots of Case-7 condition.

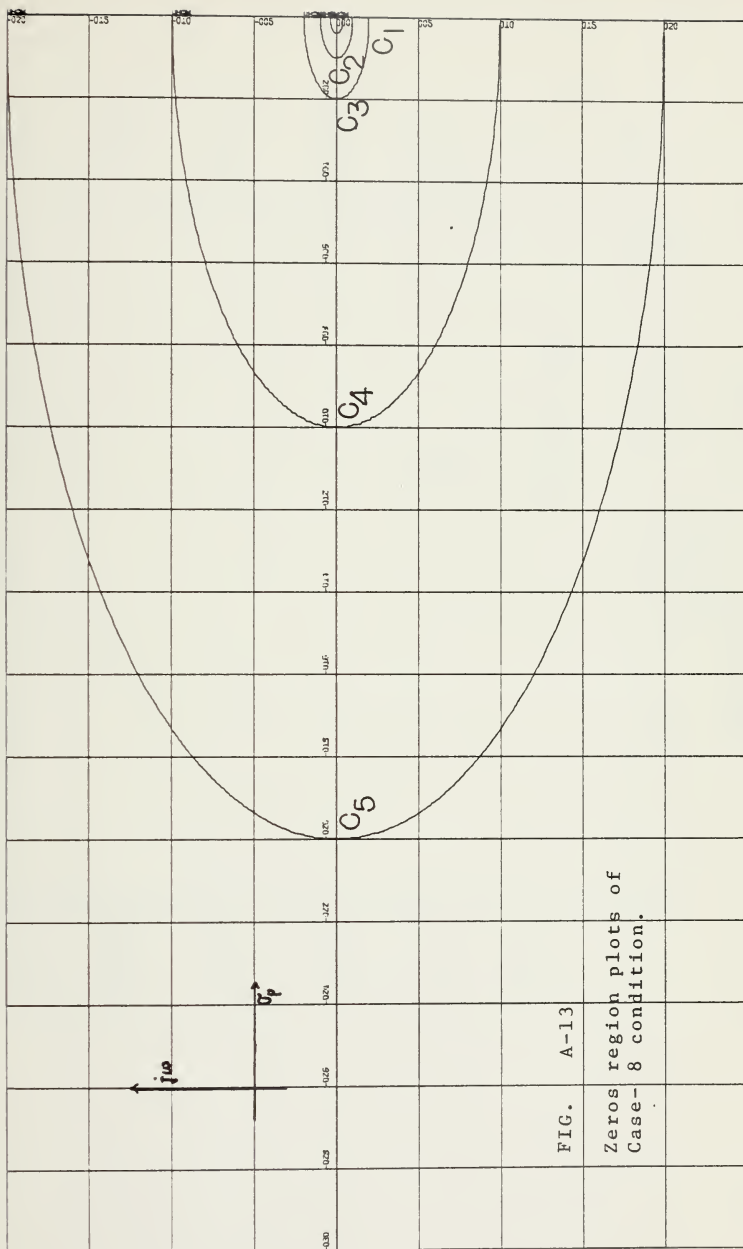


FIG. A-13

Zeros region plots of Case-8 condition.

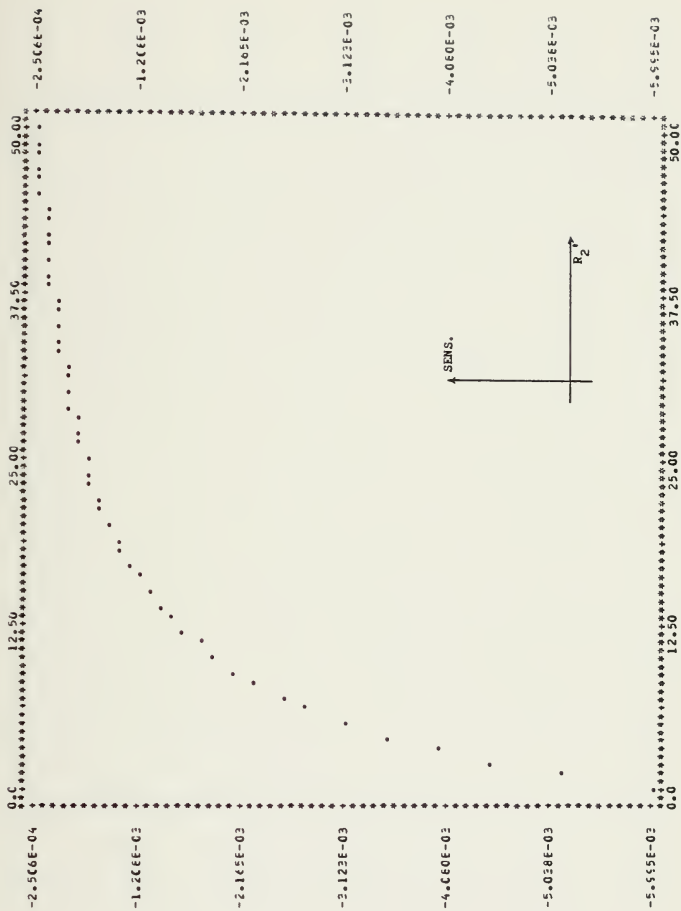
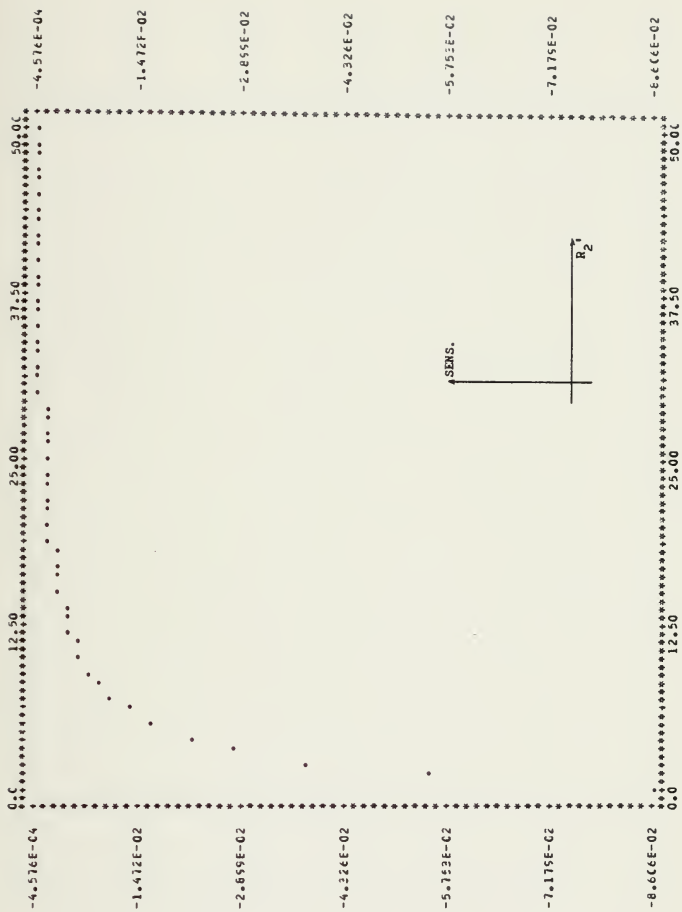


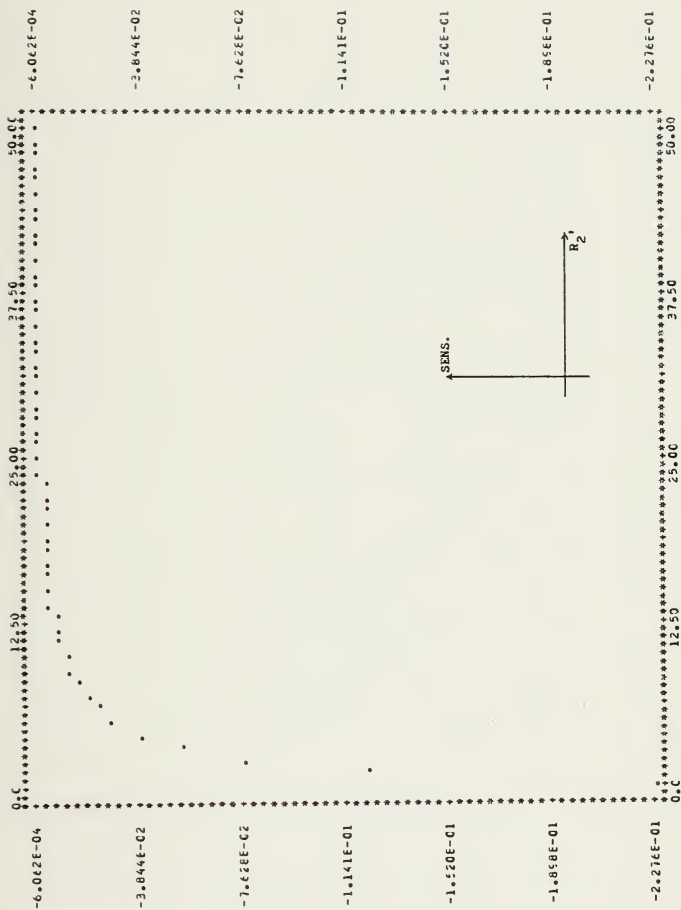
FIG B-1.

The plot of sensitivity vs  $R_2$ , is shown for given frequency 1 r./sec.



**FIG. B 2.**

Sensitivity plot vs  $R_2'$  is shown here for given  
frequency 5 rad/sec.



**FIG. B 3.**

The plot of sensitivity vs  $R_2$  is shown here  
for given frequency 10 rad/sec.

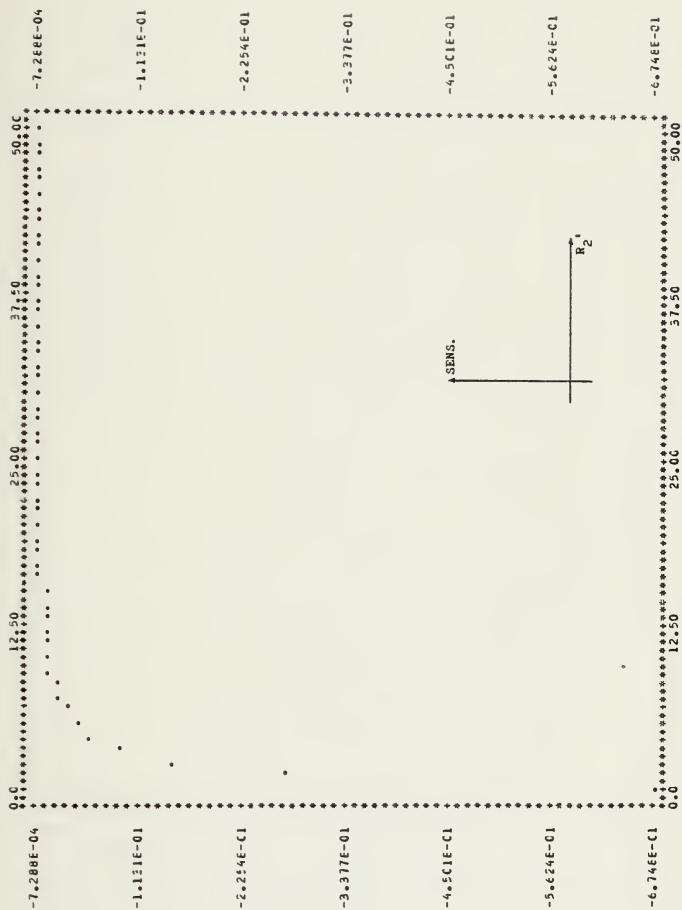
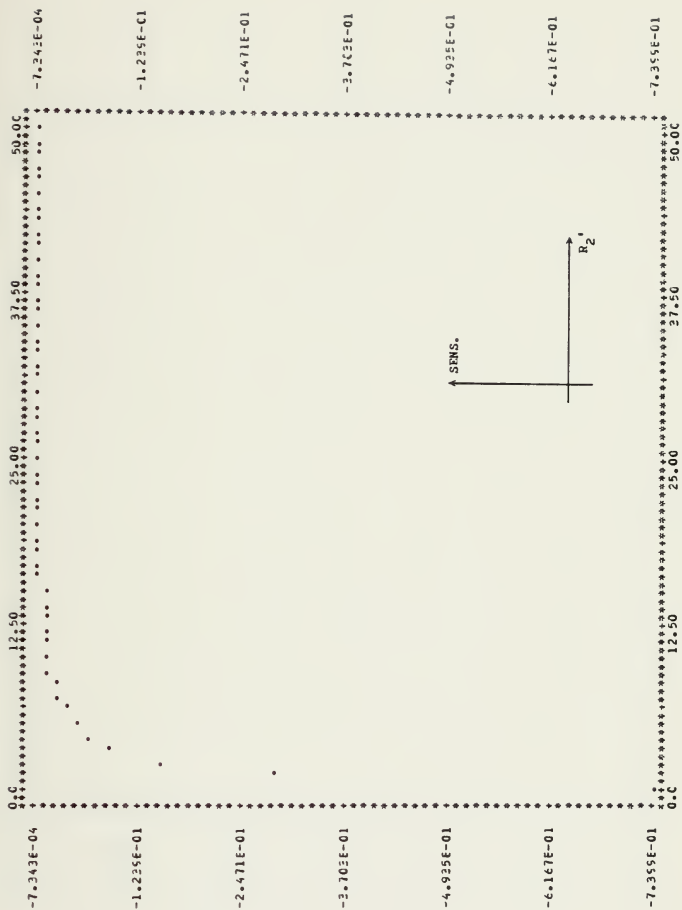
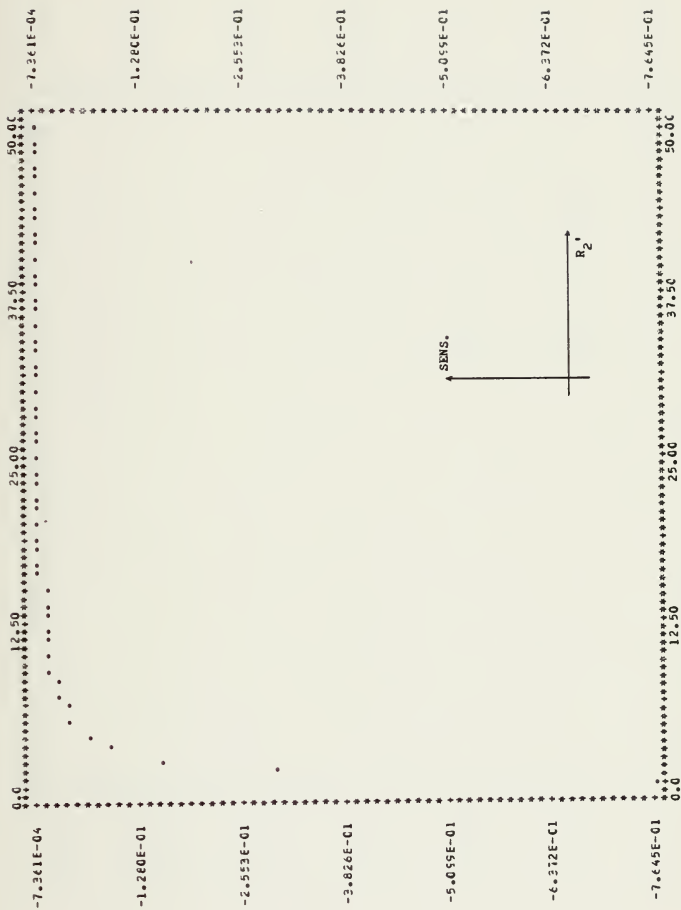


FIG. B4.

The plot of sensitivity vs  $R_2'$  is shown here for given frequency 50 rad/sec.

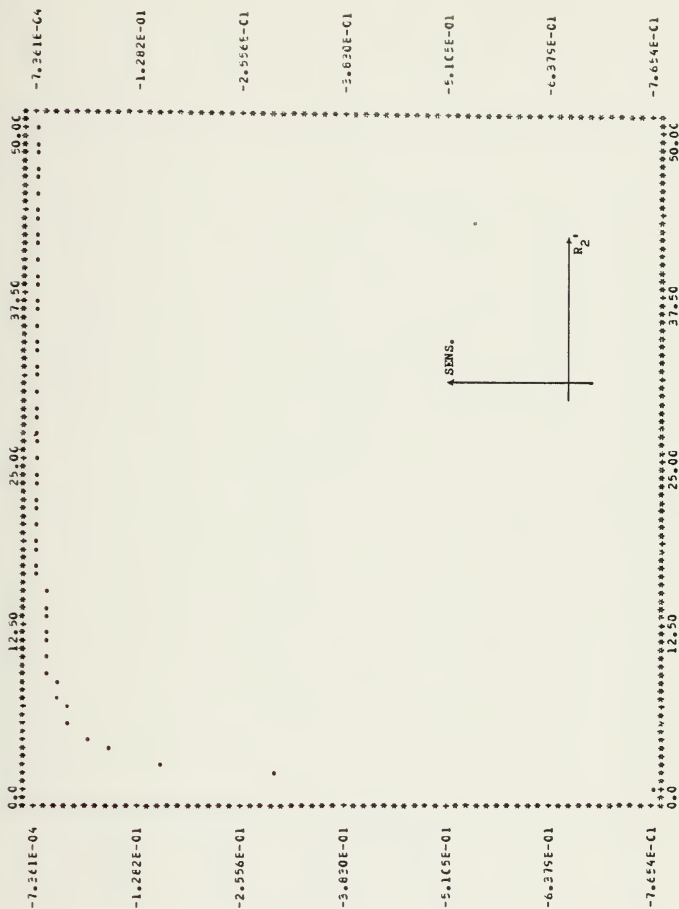


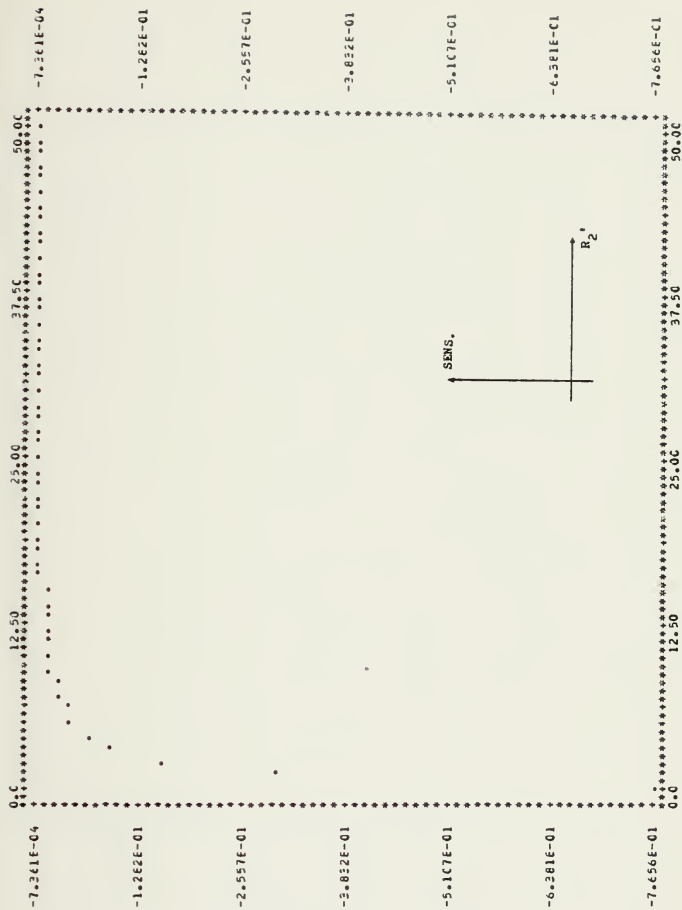
The plot of sensitivity vs  $R_1$  for given frequency  
100 rad./sec.



**FIG. B 6.**

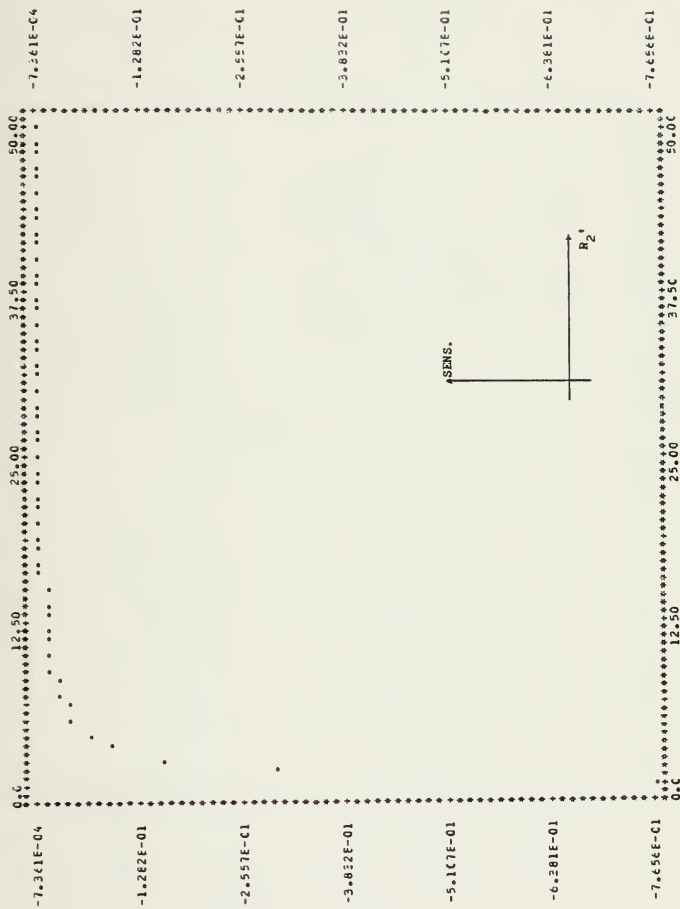
The plot of sensitivity vs  $R_2'$  is shown here for  
given frequency 500 rad/sec.





**FIG. B 8.**

The plot of sensitivity vs  $R_2$  is shown for given frequency 5000 rad/sec.



**FIG. B 9**

The plot of sensitivity vs  $R_2$  for  
given frequency 10000 rad/sec.

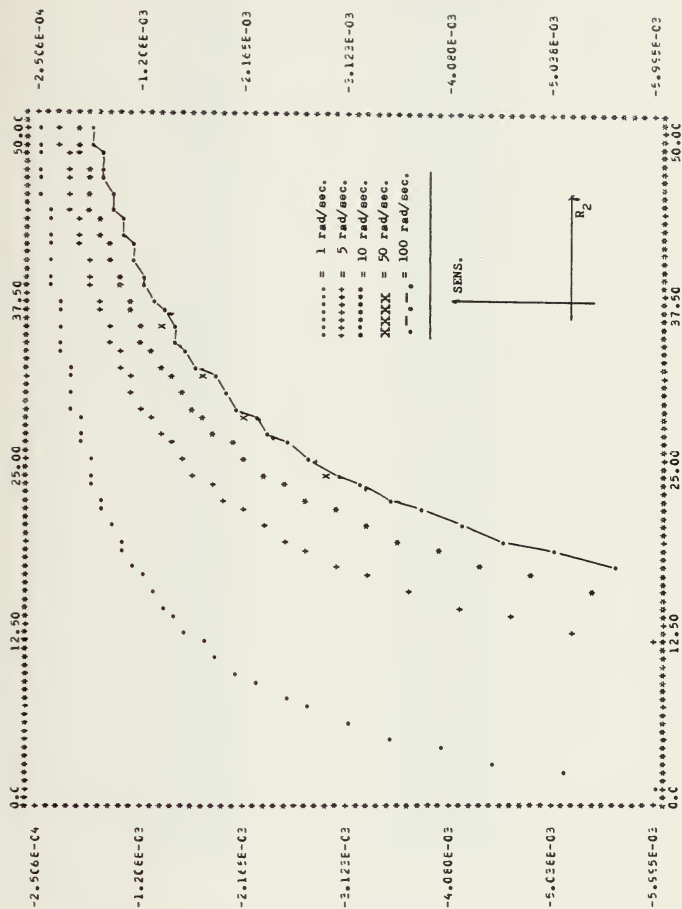


FIG. B.10.

The plots of sensitivity vs  $R_1$  are shown here for given various frequencies.

THE VALUE OF SENSITIVITY AT GIVEN  $w =$

1.000

R2S	SENSITIVITY
0.0	-0.005995
1.000	-0.005160
2.000	-0.004489
3.000	-0.003940
4.000	-0.003486
5.000	-0.003107
6.000	-0.002786
7.000	-0.002512
8.000	-0.002277
9.000	-0.002073
10.000	-0.001896
11.000	-0.001740
12.000	-0.001603
13.000	-0.001481
14.000	-0.001373
15.000	-0.001276
16.000	-0.001189
17.000	-0.001111
18.000	-0.001040
19.000	-0.000976
20.000	-0.000917
21.000	-0.000864
22.000	-0.000815
23.000	-0.000770
24.000	-0.000729
25.000	-0.000691
26.000	-0.000656
27.000	-0.000623
28.000	-0.000593
29.000	-0.000565
30.000	-0.000539
31.000	-0.000515
32.000	-0.000492
33.000	-0.000471
34.000	-0.000451
35.000	-0.000432
36.000	-0.000415
37.000	-0.000398
38.000	-0.000383
39.000	-0.000368
40.000	-0.000354
41.000	-0.000341
42.000	-0.000329
43.000	-0.000317
44.000	-0.000306
45.000	-0.000296
46.000	-0.000286
47.000	-0.000276
48.000	-0.000267
49.000	-0.000259
50.000	-0.000251

TABLE B - 1.

THE VALUE OF SENSITIVITY AT GIVEN W =

5.000

RZS	SENSITIVITY
0.0000	-0.086060
0.1000	-0.0834744
0.2000	-0.0803378
0.3000	-0.0767736
0.4000	-0.0728188
0.5000	-0.0685711
0.6000	-0.0640351
0.7000	-0.0592115
0.8000	-0.0541365
0.9000	-0.0488722
1.0000	-0.0434668
1.1000	-0.0379799
1.2000	-0.0325252
1.3000	-0.0271445
1.4000	-0.0218718
1.5000	-0.0167354
1.6000	-0.0117441
1.7000	-0.0069170
1.8000	-0.0022334
1.9000	-0.0023226
2.0000	-0.002143
2.1000	-0.0019817
2.2000	-0.001837
2.3000	-0.001707
2.4000	-0.001591
2.5000	-0.001487
2.6000	-0.001392
2.7000	-0.001306
2.8000	-0.001228
2.9000	-0.001157
3.0000	-0.001091
3.1000	-0.001031
3.2000	-0.000976
3.3000	-0.000925
3.4000	-0.000878
3.5000	-0.000835
3.6000	-0.000795
3.7000	-0.000757
3.8000	-0.000722
3.9000	-0.000690
4.0000	-0.000659
4.1000	-0.000631
4.2000	-0.000604
4.3000	-0.000579
4.4000	-0.000556
4.5000	-0.000534
4.6000	-0.000513
4.7000	-0.000494
4.8000	-0.000475
4.9000	-0.000458

TABLE B -2.

THE VALUE OF SENSITIVITY AT GIVEN W = 10.000

R <sup>2</sup> S	SENSITIVITY
0.0000	-0.227638
1.0000	-0.123034
2.0000	-0.076772
3.0000	-0.052258
4.0000	-0.037541
5.0000	-0.028735
6.0000	-0.022255
7.0000	-0.018007
8.0000	-0.014809
9.0000	-0.012442
10.0000	-0.010540
11.0000	-0.009092
12.0000	-0.007853
13.0000	-0.006855
14.0000	-0.006082
15.0000	-0.005412
16.0000	-0.004846
17.0000	-0.004386
18.0000	-0.003935
19.0000	-0.003595
20.0000	-0.003264
21.0000	-0.002942
22.0000	-0.002712
23.0000	-0.002477
24.0000	-0.002250
25.0000	-0.002031
26.0000	-0.001820
27.0000	-0.001615
28.0000	-0.001422
29.0000	-0.001240
30.0000	-0.001079
31.0000	-0.000936
32.0000	-0.000801
33.0000	-0.000682
34.0000	-0.000575
35.0000	-0.000485
36.0000	-0.000401
37.0000	-0.000324
38.0000	-0.000252
39.0000	-0.000185
40.0000	-0.000126
41.0000	-0.000070
42.0000	-0.000018
43.0000	-0.000056
44.0000	-0.000024
45.0000	-0.000008
46.0000	-0.000002
47.0000	-0.000000
48.0000	-0.000000
49.0000	-0.000000
50.0000	-0.000000

TABLE B - 3

THE VALUE OF SENSITIVITY AT GIVEN W = 50.000

R2S	SENSITIVITY
C.C	-0.674758
1.0000	-0.270416
2.0000	-0.144071
3.0000	-0.085186
4.0000	-0.060163
5.0000	-0.043783
6.0000	-0.033117
7.0000	-0.025519
8.0000	-0.020063
9.0000	-0.017112
10.0000	-0.014305
11.0000	-0.012135
12.0000	-0.010423
13.0000	-0.009000
14.0000	-0.007900
15.0000	-0.007000
16.0000	-0.006233
17.0000	-0.005587
18.0000	-0.005033
19.0000	-0.004557
20.0000	-0.004146
21.0000	-0.003788
22.0000	-0.003475
23.0000	-0.003195
24.0000	-0.002954
25.0000	-0.002737
26.0000	-0.002542
27.0000	-0.002368
28.0000	-0.002211
29.0000	-0.002066
30.0000	-0.001921
31.0000	-0.001784
32.0000	-0.001717
33.0000	-0.001619
34.0000	-0.001530
35.0000	-0.001447
36.0000	-0.001371
37.0000	-0.001301
38.0000	-0.001237
39.0000	-0.001176
40.0000	-0.001121
41.0000	-0.001069
42.0000	-0.001020
43.0000	-0.000975
44.0000	-0.000933
45.0000	-0.000893
46.0000	-0.000856
47.0000	-0.000821
48.0000	-0.000786
49.0000	-0.000758
50.0000	-0.000729

TABLE B - 4

THE VALUE OF SENSITIVITY AT GIVEN  $W = 100.000$

R2S	SENSITIVITY
0.0	-0.739854
1.0000	-0.284703
2.0000	-0.149352
3.0000	-0.091353
4.0000	-0.061453
5.0000	-0.044653
6.0000	-0.033354
7.0000	-0.026652
8.0000	-0.021151
9.0000	-0.017155
10.0000	-0.014455
11.0000	-0.012255
12.0000	-0.010655
13.0000	-0.009551
14.0000	-0.008953
15.0000	-0.008056
16.0000	-0.007654
17.0000	-0.006555
18.0000	-0.005558
19.0000	-0.004653
20.0000	-0.004157
21.0000	-0.003355
22.0000	-0.002955
23.0000	-0.002555
24.0000	-0.002251
25.0000	-0.002052
26.0000	-0.001855
27.0000	-0.001655
28.0000	-0.001450
29.0000	-0.001257
30.0000	-0.001155
31.0000	-0.001055
32.0000	-0.000951
33.0000	-0.000853
34.0000	-0.000752
35.0000	-0.000659
36.0000	-0.000583
37.0000	-0.000512
38.0000	-0.000447
39.0000	-0.000386
40.0000	-0.000330
41.0000	-0.000274
42.0000	-0.000228
43.0000	-0.000183
44.0000	-0.000140
45.0000	-0.000100
46.0000	-0.000066
47.0000	-0.000038
48.0000	-0.000015
49.0000	-0.000007
50.0000	-0.000003

TABLE B - 5

THE VALUE OF SENSITIVITY AT GIVEN W = 500.000

R2S	SENSITIVITY
0.0000	-0.764546
1.0000	-0.2899726
2.0000	-0.1511758
3.0000	-0.092291
4.0000	-0.062240
5.0000	-0.0444933
6.0000	-0.033355
7.0000	-0.025655
8.0000	-0.0217433
9.0000	-0.017441
10.0000	-0.014533
11.0000	-0.0122444
12.0000	-0.0106995
13.0000	-0.0095933
14.0000	-0.0080533
15.0000	-0.007112
16.0000	-0.00627
17.0000	-0.00555
18.0000	-0.004901
19.0000	-0.00438
20.0000	-0.003900
21.0000	-0.00347
22.0000	-0.00305
23.0000	-0.00268
24.0000	-0.00230
25.0000	-0.00190
26.0000	-0.00153
27.0000	-0.00126
28.0000	-0.00107
29.0000	-0.00093
30.0000	-0.00080
31.0000	-0.00069
32.0000	-0.00059
33.0000	-0.00050
34.0000	-0.00042
35.0000	-0.00035
36.0000	-0.00029
37.0000	-0.00024
38.0000	-0.00019
39.0000	-0.00015
40.0000	-0.00011
41.0000	-0.00008
42.0000	-0.00006
43.0000	-0.00004
44.0000	-0.00003
45.0000	-0.00002
46.0000	-0.00001
47.0000	-0.00000
48.0000	-0.00000
49.0000	-0.00000
50.0000	-0.00000

TABLE B - 6

THE VALUE OF SENSITIVITY AT GIVEN W = 1000.000

R2S	SENSITIVITY
0.0	-0.765334
1.0000	-0.289687
2.0000	-0.151215
3.0000	-0.092615
4.0000	-0.062245
5.0000	-0.044592
6.0000	-0.033331
7.0000	-0.026499
8.0000	-0.021266
9.0000	-0.017443
10.0000	-0.014565
11.0000	-0.012343
12.0000	-0.010651
13.0000	-0.009519
14.0000	-0.008600
15.0000	-0.007711
16.0000	-0.006932
17.0000	-0.006265
18.0000	-0.005610
19.0000	-0.005046
20.0000	-0.004530
21.0000	-0.004067
22.0000	-0.003651
23.0000	-0.003279
24.0000	-0.002951
25.0000	-0.002670
26.0000	-0.002437
27.0000	-0.002239
28.0000	-0.002077
29.0000	-0.001953
30.0000	-0.001863
31.0000	-0.001794
32.0000	-0.001736
33.0000	-0.001687
34.0000	-0.001646
35.0000	-0.001613
36.0000	-0.001586
37.0000	-0.001563
38.0000	-0.001545
39.0000	-0.001530
40.0000	-0.001519
41.0000	-0.001510
42.0000	-0.001503
43.0000	-0.001498
44.0000	-0.001493
45.0000	-0.001489
46.0000	-0.001486
47.0000	-0.001483
48.0000	-0.001481
49.0000	-0.001479
50.0000	-0.001478

TABLE B - 7

THE VALUE OF SENSITIVITY AT GIVEN W = 5000.000

R2S	SENSITIVITY
0.0000	-0.765613
0.0000	-0.289959
0.0000	-0.151223
0.0000	-0.092627
0.0000	-0.062250
0.0000	-0.044595
0.0000	-0.033933
0.0000	-0.026501
0.0000	-0.021267
0.0000	-0.017444
0.0000	-0.014566
0.0000	-0.012246
0.0000	-0.010197
0.0000	-0.008555
0.0000	-0.008054
0.0000	-0.007711
0.0000	-0.006632
0.0000	-0.005665
0.0000	-0.005102
0.0000	-0.004619
0.0000	-0.004201
0.0000	-0.003837
0.0000	-0.003515
0.0000	-0.003239
0.0000	-0.002991
0.0000	-0.002770
0.0000	-0.002573
0.0000	-0.002396
0.0000	-0.002237
0.0000	-0.002093
0.0000	-0.001963
0.0000	-0.001844
0.0000	-0.001736
0.0000	-0.001637
0.0000	-0.001547
0.0000	-0.001463
0.0000	-0.001386
0.0000	-0.001315
0.0000	-0.001250
0.0000	-0.001189
0.0000	-0.001133
0.0000	-0.001080
0.0000	-0.001031
0.0000	-0.000985
0.0000	-0.000943
0.0000	-0.000903
0.0000	-0.000865
0.0000	-0.000830
0.0000	-0.000797
0.0000	-0.000766
0.0000	-0.000736

TABLE B - 8

THE VALUE OF SENSITIVITY AT GIVEN  $W = 10000.000$

R2S	SENSITIVITY
0.0	-0.765621
1.000	-0.289540
2.0000	-0.151234
3.0000	-0.092527
4.0000	-0.062500
5.0000	-0.044595
6.0000	-0.033333
7.0000	-0.026501
8.0000	-0.021267
9.0000	-0.017444
10.0000	-0.014566
11.0000	-0.012346
12.0000	-0.010637
13.0000	-0.009195
14.0000	-0.008054
15.0000	-0.007113
16.0000	-0.006327
17.0000	-0.005665
18.0000	-0.005102
19.0000	-0.004619
20.0000	-0.004201
21.0000	-0.003857
22.0000	-0.003551
23.0000	-0.003299
24.0000	-0.003051
25.0000	-0.002817
26.0000	-0.002596
27.0000	-0.002387
28.0000	-0.002193
29.0000	-0.002013
30.0000	-0.001844
31.0000	-0.001686
32.0000	-0.001537
33.0000	-0.001396
34.0000	-0.001263
35.0000	-0.001146
36.0000	-0.001038
37.0000	-0.000935
38.0000	-0.000839
39.0000	-0.000749
40.0000	-0.000663
41.0000	-0.000580
42.0000	-0.000501
43.0000	-0.000425
44.0000	-0.000353
45.0000	-0.000285
46.0000	-0.000220
47.0000	-0.000157
48.0000	-0.000097
49.0000	-0.000039
50.0000	-0.000000

TABLE B - 9

APPENDIX B  
COMPUTER PROGRAM

A. ZEROS REGION

This program will plot the realizability region of general bi-quadratic function of the form

$$Z(s) = \frac{s^2 + 2*\text{SIGMAZ}*s + \text{OMEGAZ}^2}{s^2 + 2*\text{SIGMAP}*s + \text{OMEGAP}^2}$$

The input to this program is via punched card. One card is required for each plot. The input variables are Sigmap and Omegap specified in columns 1 - 20 as two fields of F 10.3 format.

The output is a plot of allowable region for zeros of Z(s) for a given pair of poles. The criterion for the plot is that Z(s) must be positive real.

The output may be either one (or both) of two types of plots depending on the needs of the programmers. If a fast solution is desired the "CALL DRAW" statements must be removed to enable the program to be run on QUICKRUN. The result will be a rough plot on the line printer output.

If maximum accuracy is desired a CALCOMP plot may be obtained by making a regular FORTCLG run with the "CALL DRAW" cards in the deck. In this case a line printer plot will also be generated, unless the "CALL PLOTP" cards are moved. For the CALCOMP run, "REGION.GO-100K" must be specified on the EXEC card.

The routines DRAW and PLOTP are called by this program. Both are in the MPSLIB at NPS. Neither a copy of the decks nor special job control language is needed to run this program on the NPS IBM 360.



The computer program is shown in Figure A-14.

## B. SENSITIVITY FUNCTION

This program will plot the sensitivity of general biquadratic function with respect to the changing of its components at a particular frequency.

The equation used in this program is,

$$S_{R_2}^{T(j\omega, R_2')} (j\omega, R_2') = - \frac{W (T(j\omega, R_2' = 0) - T(j\omega, R_2' = \text{INF.}))}{(W + R_2')^2}$$

where  $T(j\omega, R_2')$ ,  $W$ ,  $T(j\omega, R_2' = 0)$ ,  $T(j\omega, R_2' = \text{INF.})$  and  $R_2'$  correspond to Figure 17 in Chapter IV. E.

The output is a plot of the magnitude of the sensitivity function above versus  $R_2'$ .

The input of this program is via punched card. One card is required for each plot. The input variable is frequency in rad/sec. specified in columns 1 - 10 as one field of F 10.3 format.

The output is a rough plot on line printer output.

The routine "CALL PLOTP" is called by this program.

The computer program is shown in Figure B-11.



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